∂ 10.1 figure it out

A firm with market power faces an inverse demand curve for its product of $P = 100 - 10Q$. Assume that the firm faces a marginal cost curve of $MC = 10 + 10Q$.

a. If the firm cannot price discriminate, what is the profit-maximizing level of output and price?

b. If the firm cannot price discriminate, what are the levels of consumer and producer surplus in the market, assuming the firm maximizes its profit? Calculate the deadweight loss from market power.

c. If the firm has the ability to practice perfect price discrimination, what is the firm's output?

d. If the firm practices perfect price discrimination, what are the levels of consumer and producer surplus? What is the deadweight loss from market power?

e–h. Redo parts (a)–(d) using calculus and confirm that your answers are the same.

Solution:

a. If the firm cannot price discriminate, it maximizes profit by producing where $MR = MC$. If the inverse demand function is $P = 100 - 10Q$, then the marginal revenue must be $MR = 100 - 20Q$. (Remember that, for any linear inverse demand function $P = a - bQ$, marginal revenue is $MR = a - 2bQ$.)

Setting $MR = MC$, we obtain

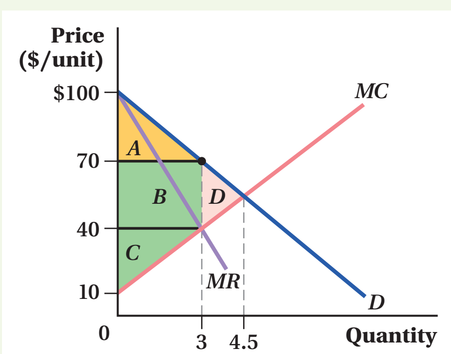
$$100 - 20Q = 10 + 10Q$$
$$90 = 30Q$$
$$Q = 3$$

To find the optimal price, we plug $Q = 3$ into the inverse demand equation:

$$\begin{aligned} P &= 100 - 10Q \\ &= 100 - 10(3) \\ &= 100 - 30 \\ &= 70 \end{aligned}$$

The firm sells 3 units at a price of \$70 each.

b. To find consumer and producer surplus, we need to start with a diagram showing the demand, marginal revenue, and marginal cost curves:



Consumer surplus is the area above price and below demand (area A). Producer surplus is the area above marginal cost but below the price (area $B + C$). (Note that we could just label these two areas as a large trapezoid, but it is easier to remember the formulas for the area of a rectangle and a triangle!) We can calculate the areas:

$$\begin{aligned}\text{Area } A &= \frac{1}{2}\text{ base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times (\$100 - \$70)\\ &= 0.5(3)(\$30)\\ &= \$45\end{aligned}$$

Consumer surplus is \$45.

$$\text{Area } B = \text{base} \times \text{height}$$

To get the height of areas B and C, we need the MC of producing a quantity of 3: $MC = 10 + 10Q = 10 + 10(3) = \$40$. So,

$$\begin{aligned}\text{Area } B &= 3 \times (\$70 - \$40)\\ &= 3(\$30)\\ &= \$90\end{aligned}$$

$$\begin{aligned}\text{Area } C &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 3 \times (\$40 - \$10)\\ &= 0.5(3)(\$30)\\ &= \$45\end{aligned}$$

So, producer surplus = Area B + Area C = \$90 + \$45 = \$135.

The deadweight loss from market power is the loss in surplus that occurs because the market is not producing the competitive quantity. To calculate the competitive quantity, we set $P = MC$:

$$\begin{aligned}100 - 10Q &= 10 + 10Q\\ 90 &= 20Q\\ Q &= 4.5\end{aligned}$$

The deadweight loss can be seen on the diagram as area D:

$$\begin{aligned}\text{Area } D &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times (4.5 - 3) \times (\$70 - \$40)\\ &= 0.5(1.5)(\$30)\\ &= \$22.50\end{aligned}$$

The deadweight loss from market power is \$22.50.

c. If the firm practices perfect price discrimination, it will produce where $P = MC$. As we saw in part (b) above, this means that the firm will produce 4.5 units.

d. If the firm practices perfect price discrimination, consumer surplus will be zero because every consumer will be charged a price equal to his willingness to pay. Producer surplus will be the full area between the demand curve and the marginal cost curve (area $A + B + C + D$):

$$\begin{aligned}\text{Producer surplus} &= \text{area } A + \text{area } B + \text{area } C + \text{area } D\\ &= \$45 + \$90 + \$45 + \$22.50\\ &= \$202.50\end{aligned}$$

There is no deadweight loss when the firm perfectly price discriminates. The competitive output level is achieved ($Q = 4.5$). Producers end up with the entire surplus available in the market.

e. The firm's inverse demand function is given as $P = 100 - 10Q$. This implies total revenue is given by $TR(Q) = P(Q) \times Q = 100Q - 10Q^2$. Marginal revenue is

$$MR = \frac{dTR(Q)}{dq} = 100 - 20Q$$

Note that without complete total cost information (for example we don't know about fixed costs), we are unable to set up a profit function to maximize directly using calculus. We can still use calculus indirectly, however, by using marginal revenue as determined using calculus above. Profit maximization requires the firm to select output such that $MR = MC$. This is

$$\begin{aligned}MR = 100 - 20Q &= 10 + 10Q = MC\\ 90 &= 30Q\\ Q &= 3\end{aligned}$$

Thus, we conclude that $Q = 3$ is the profit-maximizing level of output. The optimal price is $P = 100 - 10(3) = 70$, as in part (a).

f. Consumer surplus is

$$\begin{aligned}CS &= \textstyle\int_0^3\big((100 - 10Q) - 70\big)\,dQ\\ &= \textstyle\int_0^3(30 - 10Q)\,dQ = \int_0^3 30\,dQ - \int_0^3 10Q\,dQ\\ &= [30Q]_0^3 - \left[\frac{10Q^2}{2}\right]_0^3 = [30Q]_0^3 - [5Q^2]_0^3\\ &= [30(3) - 30(0)] - [5(3)^2 - 5(0)^2]\\ &= (90 - 0) - (45 - 0) = 45\end{aligned}$$

Producer surplus is

$$\begin{aligned}PS &= \textstyle\int_0^3\big(70 - (10 + 10Q)\big)\,dQ\\ &= \textstyle\int_0^3(60 - 10Q)\,dQ = \int_0^3 60\,dQ - \int_0^3 10Q\,dQ\\ &= [60Q]_0^3 - \left[\frac{10Q^2}{2}\right]_0^3 = [60Q]_0^3 - [5Q^2]_0^3\\ &= [60(3) - 60(0)] - [5(3)^2 - 5(0)^2]\\ &= (180 - 0) - (45 - 0) = 135\end{aligned}$$

Deadweight loss is

$$\begin{aligned}DWL &= \textstyle\int_3^{4.5}\big((100 - 10Q) - (10 + 10Q)\big)dQ = \int_3^{4.5}(90 - 20Q)dQ = \int_3^{4.5} 90\,dQ - \int_3^{4.5} 20Q\,dQ\\ &= [90Q]_3^{4.5} - \left[\frac{20Q^2}{2}\right]_3^{4.5} = [90Q]_3^{4.5} - [10Q^2]_3^{4.5}\\ &= [90(4.5) - 90(3)] - [10(4.5)^2 - 10(3)^2]\\ &= (405 - 270) - (202.5 - 90) = 22.5\end{aligned}$$

Consumer surplus, producer surplus, and deadweight loss therefore are \$45, \$135, \$22.50, respectively, just as found in part (b).

g. The reasoning for the firm choosing to produce 4.5 units is unchanged using calculus methods.

h. As the perfect price-discriminating monopolist maximizes its producer surplus (which is equal to the total surplus), we get

$$PS = \int_0^{4.5}\big((100 - 10Q) - (10 + 10Q)\big)dQ = \int_0^{4.5} (90 - 20Q)dQ = \int_0^{4.5} 90\ dQ - \int_0^{4.5} 20Q\ dQ$$
$$= [90Q]_0^{4.5} - \left[\frac{20Q^2}{2}\right]_0^{4.5} = [90Q]_0^{4.5} - [10Q^2]_0^{4.5}$$
$$= [90(4.5) - 90(0)] - [10(4.5)^2 - 10(0)^2]$$
$$= (405 - 0) - (202.5 - 0) = 202.5$$

This corresponds to \$202.50, as in part (d).

Consumer surplus is then

$$CS = \int_0^{4.5}\big((100 - 10Q) - (100 - 10Q)\big)\ dQ = \int_0^{4.5} 0\ dQ = 0$$

And, deadweight loss is similarly zero, also as in part (d):

$$DWL = \int_{4.5}^{4.5}\big((100 - 10Q) - (10 + 10Q)\big)\ dQ = \int_{4.5}^{4.5} (90 - 20Q)dQ = 0$$

Note that this integral equals zero because the integration is over a zero quantity change.