∂ 10.5 figure it out

You have been hired as an intern at the Golden Eagle Country Club Golf Course. You have been assigned the task of creating the pricing scheme for the golf course, which typically charges an annual membership fee and a per-use cost to its customers. Each of your customers is estimated to have the following demand curve for rounds of golf per year:

$$Q = 300 - 5P$$

a. If Golden Eagle can provide rounds of golf at a constant marginal cost of \$50 and charges that amount per round of golf, what is the most that members would be willing to pay for the annual membership fee?

b. Suppose that the golf course decides to cancel all membership fees and to instead establish a block pricing structure with two different prices for rounds of golf. For this case, use calculus to identify the two prices the course will use to maximize producer surplus.

c. If the golf course establishes a block pricing structure with two different prices for rounds of golf and maximizes producer surplus, use calculus to identify consumer surplus, producer surplus, and total surplus.

Solution:

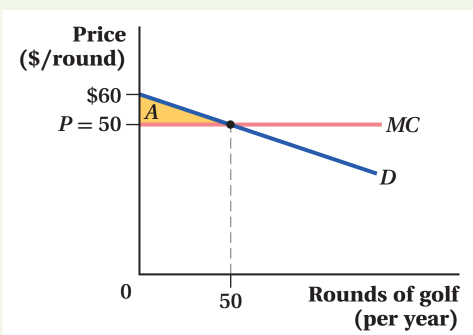
a. This pricing scheme, with an annual membership fee and a per-unit price, is a two-part tariff. If the price per round of golf is set at $P = \$50$, then each member will want to play

$$\begin{aligned} Q &= 300 - 5P \\ &= 300 - 5(50) \\ &= 300 - 250 \\ &= 50 \text{ rounds per year} \end{aligned}$$

With this knowledge, we can determine the maximum annual membership fee each customer is willing to pay. This will be equal to the amount of consumer surplus the customer will get from playing 50 rounds of golf each year at a price of \$50 per round.

To calculate consumer surplus, it is easiest to draw a diagram, plot the demand curve, and find the area of consumer surplus. To simplify matters, let's rearrange the demand function into an inverse demand function:

$$\begin{aligned} Q &= 300 - 5P \\ 5P &= 300 - Q \\ P &= 60 - 0.2Q \end{aligned}$$



The vertical intercept is 60 and the consumer surplus is the area below the demand curve and above the price of \$50, area A. We can calculate the area of triangle A:

$$\begin{aligned}\text{Area of } A &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 50 \times (\$60 - \$50) = 0.5(50)(\$10)\\ &= \$250\end{aligned}$$

If the golf course set the price of a round of golf at \$50, the consumer would purchase 50 rounds per year. This gives the golfer a consumer surplus equal to \$250. Therefore, customers would be willing to pay up to \$250 for an annual membership.

b. For a block pricing structure with two different prices, we will first decompose total revenue from each price tier. Total revenue from the high-price tier is

$$TR_1 = TR(P_1) = P_1 \times Q(P_1) = P_1(300 - 5P_1) = 300P_1 - 5P_1^2$$

Excess demand for output at the lower-price tier is

$$\begin{aligned}Q_2 &= 300 - 5P_2 - Q_1(P_1)\\ &= 300 - 5P_2 - (300 - 5P_1)\\ &= 5P_1 - 5P_2\end{aligned}$$

Total revenue from this lower-price tier is

$$TR_2 = P_2 \times Q(P_2) = P_2(5P_1 - 5P_2) = 5P_1P_2 - 5P_2^2$$

The golf course's combined total revenue function is

$$TR = 300P_1 - 5P_1^2 + 5P_1P_2 - 5P_2^2$$

The golf course's total production cost for output associated with tier 1 pricing is

$$\begin{aligned}TC_1 &= \int_0^{300-5P_1} 50\, dQ = [50Q]_0^{300-5P_1}\\ &= 50(300 - 5P_1) - 50(0) = 15{,}000 - 250P_1\end{aligned}$$

The golf course's total production cost for output associated with tier 2 pricing is

$$\begin{aligned}TC_2 &= \int_{300-5P_1}^{300-5P_1+5P_1-5P_2} 50\, dQ = \int_{300-5P_1}^{300-5P_2} 50\, dQ = [50Q]_{300-5P_1}^{300-5P_2}\\ &= 50(300 - 5P_2) - 50(300 - 5P_1)\\ &= 15{,}000 - 250P_2 - 15{,}000 + 250P_1\\ &= 250P_1 - 250P_2\end{aligned}$$

The golf course's total production cost for producing output across both pricing blocks is

$$\begin{aligned}TC &= 15{,}000 - 250P_1 + 250P_1 - 250P_2\\ &= 15{,}000 - 250P_2\end{aligned}$$

The block pricing monopolist's producer surplus is

$$PS = TR - TC = 300P_1 - 5P_1^2 + 5P_1P_2 - 5P_2^2 - 15{,}000 + 250P_2$$

The golf course's objective is to choose P_1 and P_2 that maximize PS. The relevant system of first-order conditions is

$$\begin{aligned}0 &= \frac{\partial PS}{\partial P_1} = 300 - 10P_1 + 5P_2\\ 0 &= \frac{\partial PS}{\partial P_2} = 5P_1 - 10P_2 + 250\end{aligned}$$

Using the first of these first-order conditions, we may solve for P_1. This produces

$$P_1 = 30 + 0.5P_2$$

Inserting the right-hand side of this statement for P_1 into the second of the first-order conditions and solving for P_2, we have

$$\begin{aligned} 0 &= 5(30 + 0.5P_2) - 10P_2 + 250 \\ 0 &= 150 + 2.5P_2 - 10P_2 + 250 \\ 7.5P_2 &= 400 \\ P_2 &\approx 53.33 \end{aligned}$$

Thus,

$$P_1 = 30 + 0.5(53.33) \approx 56.67$$

The corresponding block 1 output is

$$\begin{aligned} Q_1 &= 300 - 5P_1 \\ &= 300 - 5(56.67) \\ &= 16.65 \end{aligned}$$

The block 2 output is

$$\begin{aligned} Q_2 &= 300 - 5P_2 - Q_1(P_1) = \\ &= 300 - 5(53.33) - 16.65 \\ &= 16.7 \end{aligned}$$

c. Producer surplus is

$$\begin{aligned} PS &= 300P_1 - 5P_1^2 + 5P_1P_2 - 5P_2^2 - 15{,}000 + 250P_2 \\ &= 300(56.67) - 5(56.67)^2 + 5(56.67)(53.33) - 5(53.33)^2 - 15{,}000 + 250(53.33) \\ &\approx 166.67 \end{aligned}$$

The inverse demand curve is $P = 60 - 0.2Q$.

Consumer surplus then is

$$\begin{aligned} CS &= \int_0^{16.65} (60 - 0.2Q - 56.67)\, dQ + \int_{16.65}^{16.65+16.7} (60 - 0.2Q - 53.33)\, dQ \\ &= \int_0^{16.65} (3.33 - 0.2Q)\, dQ + \int_{16.65}^{33.35} (6.67 - 0.2Q)\, dQ \\ &= [3.33Q]_0^{16.65} - \left[\frac{0.2Q^2}{2}\right]_0^{16.65} + [6.67Q]_{16.65}^{33.35} - \left[\frac{0.2Q^2}{2}\right]_{16.65}^{33.35} \\ &= [3.33Q]_0^{16.65} - [0.1Q^2]_0^{16.65} + [6.67Q]_{16.65}^{33.35} - [0.1Q^2]_{16.65}^{33.35} \\ &= [3.33(16.65) - 3.33(0)] - [0.1(16.65)^2 - 0.1(0)^2] + [6.67(33.35) - 6.67(16.65)] \\ &\qquad - [0.1(33.35)^2 - 0.1(16.65)^2] \\ &= (55.44 - 0) - (27.72 - 0) + (222.44 - 111.06) - (111.22 - 27.72) \\ &= 55.44 - 27.72 + 111.38 - 83.5 \\ &= 55.6 \end{aligned}$$

Total surplus in the market with block pricing (with two pricing tiers) is \$166.67 + 55.6 = \$222.27.