Solution

7. a. The inverse demand is

$$P = 1{,}000 - 2(q_S + q_A)$$

The residual demand faced by Sydney is

$$P = (1{,}000 - 2q_A) - 2q_S$$

b. The residual marginal revenue is

$$MR_S = (1{,}000 - 2q_A) - 4q_S$$

c.

$$MR_S = 1{,}000 - 2q_A - 4q_S = 200 = MC$$
$$q_S = 200 - 0.5q_A$$

d. The residual marginal revenue for Adelaide is

$$MR_A = 1{,}000 - 4q_A - 2q_S$$

Thus,

$$MR_A = 1{,}000 - 4q_A - 2q_S = 200 = MC$$
$$q_A = 200 - 0.5q_S$$

e. The profit-maximizing level of output for Sydney is

$$q_S = 200 - 0.5q_A = 200 - 0.5(200 - 0.5q_S) = 100 + 0.25q_S$$
$$= 133.33$$

The profit-maximizing level of output for Adelaide is

$$q_A = 200 - 0.5q_S = 200 - 0.5(133.33) = 133.33$$

f. The output of the industry is 266.66. Thus, the price is

$$P = 1{,}000 - 2(q_S + q_A) = 1{,}000 - 2(266.66) = \$466.66$$

Both Sydney and Adelaide earn the same profit, which is equal to

$$TR - TC = (\$466.66 - \$200) \times 133.33 \approx \$35{,}556$$

Total industry profit is equal to \$71,111.11.

g. If Sydney becomes a monopolist, she would set the price so that the marginal cost equals the marginal revenue, that is,

$$MR = 1{,}000 - 4Q = 200 = MC$$
$$Q = 200$$

The price is

$$P = 1{,}000 - 2Q = 1{,}000 - 2 \times 200 = \$600$$

The profit is now

$$TR - TC = (\$600 - \$200) \times 200 = \$80{,}000$$

Therefore, the quantity sold decreases, price increases, and so does the profit for the industry as a whole.

h. First, we can examine the Cournot equilibrium. Sydney's profit function can be written as

$$\pi_S = \big(1{,}000 - 2(q_S + q_A)\big)q_S - 200q_S$$
$$= 1{,}000q_S - 2q_S^2 - 2q_Sq_A - 200q_S$$
$$= 800q_S - 2q_S^2 - 2q_Sq_A$$

Sydney's objective is to maximize π_S by choosing q_S. Her first-order condition is

$$0 = \frac{\partial \pi_S}{\partial q_S} = 800 - 4q_S - 2q_A$$

Sydney's reaction function then is

$$4q_S = 800 - 2q_A$$
$$q_S = 200 - 0.5q_A$$

Since Sydney and Adelaide have symmetric costs, we know that Adelaide's reaction function is

$$q_A = 200 - 0.5q_S$$

Solving these equations, we see that

$$q_S = 200 - 0.5(200 - 0.5q_S)$$
$$q_S = 100 + 0.25q_S$$
$$0.75q_S = 100$$
$$q_S \approx 133.33$$

By symmetry, $q_A \approx 133.33$. Price then is determined by the demand curve: $P = 1{,}000 - 2(q_J + q_A) = 1{,}000 - 2(133.33 + 133.33) = 466.68$. Profits therefore are the same as those obtained in part (f).

Next, we can examine the monopoly equilibrium using calculus. Jointly, the goal is to maximize profit, which is $\pi = TR - TC = P(Q) \times Q - TC = (1{,}000 - 2Q)Q - 200Q = 1{,}000Q - 2Q^2 - 200Q = 800Q - 2Q^2$. The first-order condition is $0 = \frac{\partial \pi}{\partial Q} = 800 - 4Q$ or $4Q = 800$ or $Q = 200$. Price then is $P = 1{,}000 - 2(200) = 600$. Profit thus is as given in part (g).