∂ 11.1 figure it out

Suppose that Squeaky Clean and Biobase are the only two producers of chlorine for swimming pools. The inverse market demand for chlorine is $P = 32 - 2Q$, where Q is measured in tons and P is dollars per ton. Assume that chlorine can be produced by either firm at a constant marginal cost of \$16 per ton and there are no fixed costs.

a. If the two firms collude and act like a monopoly, agreeing to evenly split the market, how much will each firm produce and what will the price of a ton of chlorine be? How much profit will each firm earn?

b. Does Squeaky Clean have an incentive to cheat on this agreement by producing an additional ton of chlorine? Explain.

c. Does Squeaky Clean's decision to cheat affect Biobase's profit? Explain.

d. Suppose that both firms agree to each produce 1 ton more than they were producing in part (a). How much profit will each firm earn? Does Squeaky Clean now have an incentive to cheat on this agreement by producing another ton of chlorine? Explain.

e. Redo part (a) using calculus by solving for marginal revenue directly from the revenue function that is relevant to the collusion, and confirm that your answers are the same.

Solution:

a. If the firms agree to act like a monopoly, they will set $MR = MC$ to solve for the profit-maximizing output:

$$MR = MC$$
$$32 - 4Q = 16$$
$$4Q = 16$$

$Q = 4$ and each firm will produce 2 tons. To find the price, we substitute the market quantity ($Q = 4$) into the inverse demand equation:

$$P = 32 - 2Q = 32 - 2(4) = \$24 \text{ per ton}$$

Each firm will earn a profit of $(\$24 - \$16) \times 2 = \$16$.

b. If Squeaky Clean cheats and produces 3 tons, Q rises to 5 and price falls to \$22. Squeaky Clean's profit will be equal to $(\$22 - \$16) \times 3 = \$18$. Therefore, Squeaky Clean does have an incentive to cheat on the agreement because its profit would rise.

c. If Squeaky Clean cheats, the price in the market falls to \$22. This reduces Biobase's profit, which is now $(\$22 - \$16) \times 2 = \$12$.

d. If both firms agree to limit production to 3 tons, $Q = 6$ and $P = \$20$. Therefore, each firm earns a profit of $(\$20 - \$16) \times 3 = \$12$. If Squeaky Clean tries to produce 4 tons of chlorine, Q rises to 7 and P falls to \$18. Therefore, Squeaky Clean's profit will be $(\$18 - \$16) \times 4 = \$8$. Thus, Squeaky Clean does not have an incentive to cheat on this agreement because its profit would fall.

e. The collusion's total revenue function is

$$\begin{aligned} R(Q) &= P(Q) \times Q \\ &= (32 - 2Q)Q \\ &= 32Q - 2Q^2 \end{aligned}$$

Marginal revenue is $\frac{dR}{dQ} = 32 - 4Q$. Notice that this is the same marginal revenue as noted in part (a) and that the rest of the calculations are the same, so $Q = 4$ and $P = \$24$ per ton as in the text problem.

Alternately, we could set up the profit function by noting that marginal cost is constant and there are no fixed costs and therefore $TC = 16Q$. With this information, the profit function is

$$\begin{aligned} \pi &= R(Q) - TC \\ &= 32Q - 2Q^2 - 16Q \\ &= 16Q - 2Q^2 \end{aligned}$$

and maximize this with respect to quantity by taking the derivative with respect to Q and setting it equal to zero to form the collusion's first-order condition. Here, $\frac{d\pi}{dQ} = 16 - 4Q = 0$, so $Q = 4$ as noted above.