11.2 figure it out

OilPro and GreaseTech are the only two firms who provide oil changes in a local market in a Cournot duopoly. The oil changes performed by the two firms are identical, and consumers are indifferent about which firm they will purchase an oil change from. The market inverse demand for the oil changes is $P = 100 - 2Q$, where Q is the total number of oil changes (in thousands per year) produced by the two firms, $q_O + q_G$. OilPro has a marginal cost of $12 per oil change, while GreaseTech has a marginal cost of $20. Assume that neither firm has any fixed cost.

a. Determine each firm's reaction curve and graph it.

b. How many oil changes will each firm produce in Cournot equilibrium?

c. What will the market price for an oil change be?

d. How much profit does each firm earn?

e. Solve for the OilPro and GreaseTech's reaction functions using calculus and show that these are the same as in part (a).

Solution:

a. Start by substituting $Q = q_O + q_G$ into the market inverse demand curve:

$$P = 100 - 2Q = 100 - 2(q_O + q_G) = 100 - 2q_O - 2q_G$$

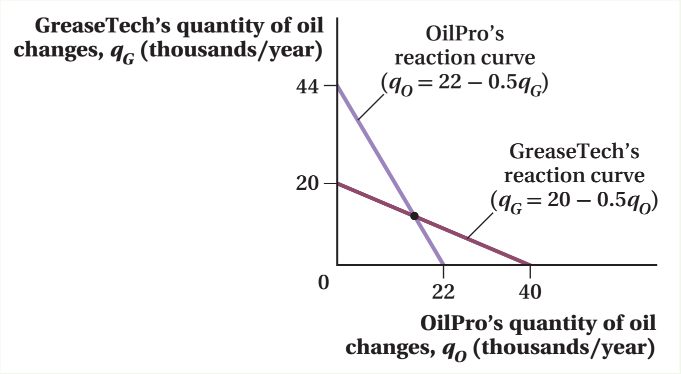
From this inverse demand curve, we can derive each firm's marginal revenue curve:

$$MR_O = 100 - 4q_O - 2q_G$$
$$MR_G = 100 - 2q_O - 4q_G$$

Each firm will set its marginal revenue equal to its marginal cost to maximize profit. From this, we can obtain each firm's reaction curve:

$$MR_O = 100 - 4q_O - 2q_G = 12$$
$$4q_O = 88 - 2q_G$$
$$q_O = 22 - 0.5q_G$$
$$MR_G = 100 - 2q_O - 4q_G = 20$$
$$4q_G = 80 - 2q_O$$
$$q_G = 20 - 0.5q_O$$

These reaction curves are shown in the figure below.



b. To solve for equilibrium, we need to substitute one firm's reaction curve into the reaction curve for the other firm:

$$q_O = 22 - 0.5q_G$$
$$q_O = 22 - 0.5(20 - 0.5q_O) = 22 - 10 + 0.25q_O = 12 + 0.25q_O$$
$$0.75q_O = 12$$
$$q_O = 16$$
$$q_G = 20 - 0.5q_O = 20 - 0.5(16) = 20 - 8 = 12$$

Therefore, OilPro produces 16,000 oil changes per year, while GreaseTech produces 12,000.

c. We can use the market inverse demand curve to determine the market price:

$$P = 100 - 2Q = 100 - 2(q_O + q_G) = 100 - 2(16 + 12) = 100 - 56 = 44$$

The price will be $44 per oil change.

d. OilPro sells 16,000 oil changes at a price of $44 for a total revenue $TR =$ 16,000 × $44 = $704,000. Total cost $TC =$ 16,000 × $12 = $192,000. Therefore, profit for OilPro is $\pi =$ $704,000 – $192,000 = $512,000.

GreaseTech sells 12,000 oil changes at a price of $44 for a total revenue of $TR =$ 12,000 × $44 = $528,000. Total cost $TC =$ 12,000 × $20 = $240,000. Thus, GreaseTech's profit is $\pi =$ $528,000 – $240,000 = $288,000.

Note that the firm with the lower marginal cost produces more output and earns a greater profit.

e. OilPro's profit function can be written as

$$\begin{aligned}\pi_O &= \big(100 - 2(q_O + q_G)\big)q_O - 12q_O \\ &= 100q_O - 2q_O^2 - 2q_Oq_G - 12q_O \\ &= 88q_O - 2q_O^2 - 2q_Oq_G\end{aligned}$$

OilPro's objective is to maximize π_O by choosing q_O. The firm's first-order condition is

$$0 = \frac{\partial \pi_O}{\partial q_O} = 88 - 4q_O - 2q_G$$

OilPro's reaction function then is

$$\begin{aligned}4q_O &= 88 - 2q_G \\ q_O &= 22 - 0.5q_G\end{aligned}$$

GreaseTech's profit function can be written as

$$\begin{aligned}\pi_G &= \big(100 - 2(q_O + q_G)\big)q_G - 20q_G \\ &= 100q_G - 2q_G^2 - 2q_Oq_G - 20q_G \\ &= 80q_G - 2q_G^2 - 2q_Oq_G\end{aligned}$$

GreaseTech's objective is to maximize π_G by choosing q_G. The firm's first-order condition is

$$0 = \frac{\partial \pi_G}{\partial q_G} = 80 - 4q_G - 2q_O$$

GreaseTech's reaction function then is

$$\begin{aligned}4q_G &= 80 - 2q_O \\ q_G &= 20 - 0.5q_G\end{aligned}$$

Since these reaction curves are the same as in part (a) of the problem, all subsequent analysis is the same and the final solution using calculus is the same as that from using algebraic methods as in the text.