∂ 11.4 figure it out

Consider our example of the two snowboard manufacturers, Burton and K2. We just determined that at the Nash equilibrium for these two firms, each firm produced 600 snowboards at a price of $300 per board. Now let's suppose that Burton launches a successful advertising campaign to convince snowboarders that its product is superior to K2's so that the demand for Burton snowboards rises to $q_B = 1{,}000 - 1.5p_B + 1.5p_K$, while the demand for K2 boards falls to $q_K = 800 - 2p_K + 0.5p_B$. (For simplicity, assume that the marginal cost is still zero for both firms.)

a. Derive each firm's reaction curve.

b. What happens to each firm's optimal price?

c. What happens to each firm's optimal output?

d. Draw the reaction curves in a diagram and indicate the equilibrium.

e. Solve for the Burton and K2's price reaction functions using calculus and show that these are the same as in part (a).

Solution:

a. To determine the firms' reaction curves, we first need to solve for each firm's marginal revenue curve:

$$MR_B = 1{,}000 - 3p_B + 1.5p_K$$
$$MR_K = 800 - 4p_K + 0.5p_B$$

By setting each firm's marginal cost equal to marginal revenue, we can find the firm's reaction curve:

$$MR_B = 1{,}000 - 3p_B + 1.5p_K = 0$$
$$3p_B = 1{,}000 + 1.5p_K$$
$$p_B = 333.33 + 0.5p_K$$
$$MR_K = 800 - 4p_K + 0.5p_B = 0$$
$$4p_K = 800 + 0.5p_B$$
$$p_K = 200 + 0.125p_B$$

b. We can solve for the equilibrium by substituting one firm's reaction curve into the other's:

$$p_B = 333.33 + 0.5p_K$$
$$p_B = 333.33 + 0.5(200 + 0.125p_B) = 333.33 + 100 + 0.0625p_B$$
$$p_B = 433.33 + 0.0625p_B$$
$$0.9375p_B = 433.33$$
$$p_B = \$462.22$$

We can then substitute p_B back into the reaction function for K2 to get the K2 price:

$$p_K = 200 + 0.125p_B$$
$$= 200 + 0.125(462.22) = 200 + 57.78 = \$257.78$$

So, the successful advertising campaign means that Burton can increase its price from the original equilibrium price of $300 (which we determined in our initial analysis of this market) to $462.22, while K2 will have to lower its own price from $300 to $257.78.

c. To find each firm's optimal output, we need to substitute the firms' prices into the inverse demand curves for each firm's product. For Burton,

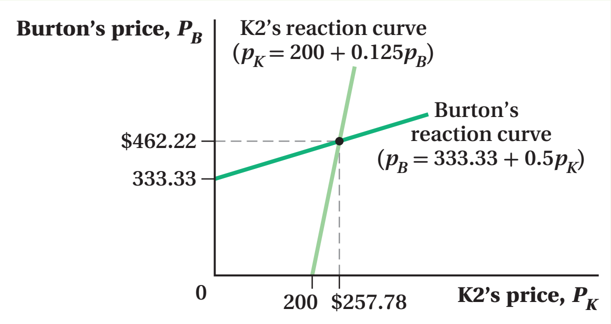
$$q_B = 1{,}000 - 1.5p_B + 1.5p_K = 1{,}000 - 1.5(462.22) + 1.5(257.78)$$
$$= 1{,}000 - 693.33 + 386.67 = 693.34$$

For K2,

$$q_K = 800 - 2p_K + 0.5p_B = 800 - 2(257.78) + 0.5(462.22) = 800 - 515.56 + 231.11 = 515.55$$

Burton now produces more snowboards (693.34 instead of 600), while K2 produces fewer (515.55 instead of 600).

d. The reaction curves are shown in the diagram below:



e. Burton's profit function can be written (which is equivalent to its revenue function since marginal cost is assumed to be zero):

$$\pi_B = (1{,}000 - 1.5p_B + 1.5p_K)p_B$$
$$= 1{,}000p_B - 1.5p_B^2 + 1.5p_Bp_K$$

Burton's objective is to maximize π_B by choosing p_B. The firm's first-order condition is

$$0 = \frac{\partial \pi_B}{\partial p_B} = 1{,}000 - 3p_B + 1.5p_K$$

Burton's reaction function then is

$$3p_B = 1{,}000 + 1.5p_K$$
$$p_B = 333.33 + 0.5p_K$$

K2's profit function can be written (which also is equivalent to its revenue function since marginal cost is assumed to be zero):

$$\pi_K = (800 - 2p_K + 0.5p_B)p_K$$
$$= 800p_K - 2p_K^2 + 0.5p_Bp_K$$

K2's objective is to maximize π_K by choosing p_K. The firm's first-order condition is

$$0 = \frac{\partial \pi_K}{\partial p_K} = 800 - 4p_K + 0.5p_B$$

K2's reaction function then is

$$4p_K = 800 + 0.5p_B$$
$$p_K = 200 + 0.125p_B$$

Since these reaction curves are the same as in part (a) of the problem, all subsequent analysis is the same and the final solution using calculus is the same as that from using algebraic methods as in the text.