∂ 2.5 figure it out

The demand for gym memberships in a small rural community is $Q = 360 - 2P$, where Q is the number of monthly members and P is the monthly membership rate.

a. Calculate the price elasticity of demand for gym memberships when the price is $50 per month.

b. Calculate the price elasticity of demand for gym memberships when the price is $100 per month.

c. Based on your answers to (a) and (b), what can you tell about the relationship between price and the price elasticity of demand along a linear demand curve?

d. Redo part (a) using calculus and confirm that your answer is the same as what you determined algebraically in part (a).

e. Redo part (b) using calculus and confirm that your answer is the same as what you determined algebraically in part (b).

f. Using the calculus version of the price elasticity of demand formula, determine the price and quantity at which total expenditure is maximized.

Solution:

a. The price elasticity of demand is calculated as

$$E = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Let's first calculate the slope of the demand curve. The easiest way to do this is to rearrange the equation in terms of P to find the inverse demand curve:

$$Q = 360 - 2P$$
$$2P = 360 - Q$$
$$P = 180 - 0.5Q$$

We can see that the slope of this demand curve is –0.5. We know this because every time Q rises by 1, P falls by 0.5.

So we know the slope and the price. To compute the elasticity, we need to know the quantity demanded at a price of $50. To find this, we plug $50 into the demand equation for P:

$$Q = 360 - 2P = 360 - 2(50) = 360 - 100 = 260$$

Now we are ready to compute the elasticity:

$$E = \frac{1}{-0.5} \cdot \frac{50}{260} = \frac{50}{-130} = -0.385$$

b. When the price is $100 per month, the quantity demanded is

$$Q = 360 - 2P = 360 - 2(100) = 360 - 200 = 160$$

Plugging into the elasticity formula, we get

$$E = \frac{1}{-0.5} \cdot \frac{100}{160} = \frac{100}{-80} = -1.25$$

c. From (a) and (b), we can see that as the price rises along a linear demand curve, demand moves from being inelastic ($0.385 < 1$) to elastic ($1.25 > 1$).

d. Using derivatives, the price elasticity of demand is

$$E^D = \frac{\partial Q^D}{\partial P} \frac{P}{Q^D}$$
$$= -2\frac{50}{260} \approx -0.385$$

e. Using derivatives, the price elasticity of demand is

$$E^D = \frac{\partial Q^D}{\partial P} \frac{P}{Q^D}$$
$$= -2\frac{100}{160} = -1.25$$

f. The online appendix explains how total expenditure is maximized when the price elasticity of demand is exactly unit-elastic. This is true when

$$E^D = \frac{\partial Q^D}{\partial P} \frac{P}{Q^D}$$
$$= -2\frac{P}{Q^D} = -1$$

Rearranging, we can see that this holds when $2P = Q^D$. Substituting into the demand equation gives

$$Q^D = 360 - 2P$$
$$2P = 360 - 2P$$
$$4P = 360$$
$$P = 90$$
$$Q^D = 360 - 2(90) = 180$$