∂ 2.1 figure it out

Suppose that the demand and supply curves for a monthly cell phone plan with unlimited texts can be represented by

$$Q^D = 50 - 0.5P$$
$$Q^S = -25 + P$$

The current price of these plans in the market is $40 per month.

a. Is this market in equilibrium? Would you expect the price to rise or fall? If so, by how much? Explain.

b. Suppose that a monthly Internet plan with price P_c is a complement to the monthly cell phone plan with unlimited texts. The expanded demand curve for the monthly cell phone plan with text messaging then is $Q^D = 75 - 0.5P - 5P_c + 0.02I$, where I is income. Suppose that the current price of monthly Internet is $25 and that monthly income (which also enters demand) is $5,000. Show that this demand curve is consistent with the demand relationship in part (a).

c. Show that the law of demand holds using calculus.

d. Show that monthly Internet plans are complementary to monthly cell phone plans using calculus.

e. Show that the law of supply holds using calculus.

Solution:

a. There are two ways to solve the first question about whether the price will rise or fall. The first is to calculate the quantity demanded and quantity supplied at the current market price of $40 to see how they compare:

$$Q^D = 50 - 0.5P = 50 - 0.5(40) = 50 - 20 = 30$$
$$Q^S = -25 + P = -25 + 40 = 15$$

Because quantity demanded is greater than quantity supplied, we can tell that there is excess demand (a shortage) in the market. Many people are trying to get texting plans, but are finding them sold out because few suppliers want to sell at that price. Prices will rise to equalize quantity supplied and quantity demanded, moving the market to equilibrium.

Alternatively, we could start by solving for the market equilibrium price:

$$Q^D = Q^S$$
$$50 - 0.5P = -25 + P$$
$$1.5P = 75$$
$$P = \$50$$

The current market price, $40, is below the market equilibrium price of $50. (This is why there is excess demand in the market.) Therefore, we would expect the price to rise by $10. When the market reaches equilibrium at a price of $50, all buyers can find sellers and all sellers can find buyers. The price will then remain at $50 unless the market changes and the demand curve or supply curve shifts.

b. Substituting $P_c = 25$ and $I = 5{,}000$ into the expanded demand curve, we can see that

$$Q^D = 75 - 0.5P - 5(25) + 0.02(5{,}000)$$
$$= 50 - 0.5P$$

This is the demand curve as given in part (a).

c. Since $\frac{\partial Q^D}{\partial P} = -0.5 < 0$, the law of demand holds. Note that this can be derived from either the regular or expanded demand curve.

d. Since $\frac{\partial Q^D}{\partial P_c} = -5 < 0$, monthly Internet is a complement to monthly cell phone plans.

e. Using the supply curve as given in the problem, $\frac{\partial Q^S}{\partial P} = 1 > 0$, so the law of supply holds.