∂ 3.1 figure it out

The demand and supply curves for newspapers in a Midwestern city are given by

$$Q^D = 152 - 20P$$
$$Q^S = 188P - 4$$

where Q is measured in thousands of newspapers per day and P in dollars per newspaper.

a. Find the equilibrium price and quantity.

b. Calculate the consumer and producer surplus at the equilibrium price.

c. Recalculate part (b) using calculus and confirm (within rounding) that your answer is the same as in part (b).

Solution:

a. Equilibrium occurs where $Q^D = Q^S$. Therefore, we can solve for equilibrium by equating the demand and supply curves:

$$Q^D = Q^S$$
$$152 - 20P = 188P - 4$$
$$156 = 208P$$
$$P = \$0.75$$

Therefore, the equilibrium price of a paper is \$0.75. To find the equilibrium quantity, we need to plug the equilibrium price into either the demand or supply curve:

$$\begin{aligned} Q^D &= 152 - 20P \\ &= 152 - 20(0.75) \\ &= 152 - 15 \\ &= 137 \end{aligned} \qquad \begin{aligned} Q^S &= 188P - 4 \\ &= 188(0.75) - 4 \\ &= 141 - 4 \\ &= 137 \end{aligned}$$

Remember that Q is measured in terms of thousands of papers each day, so the equilibrium quantity is 137,000 papers each day.

b. To calculate consumer and producer surplus, it is easiest to use a graph. First, we need to plot the demand and supply curves. For each curve, we can identify two points. The first point is the equilibrium, given by the combination of equilibrium price (\$0.75) and equilibrium quantity (137). The second point we can identify is the choke price for demand and supply. These can be determined by setting Q^D and Q^S equal to zero and solving for P:

$$\begin{aligned} Q^D &= 152 - 20P \\ 0 &= 152 - 20P \\ 20P &= 152 \\ P &= 7.6 \end{aligned} \qquad \begin{aligned} Q^S &= 188P - 4 \\ 0 &= 188P - 4 \\ 4 &= 188P \\ P &= 0.02 \end{aligned}$$

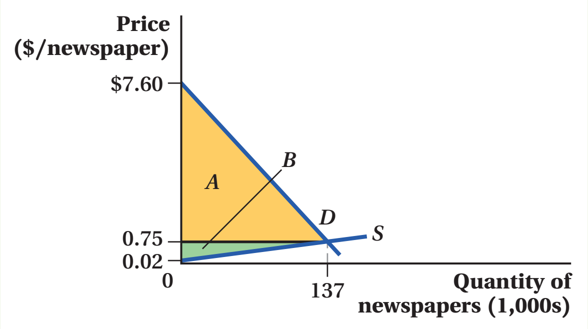
So the demand choke price is \$7.60 and the supply choke price is \$0.02.

The demand and supply curves are graphed in the figure on the next page. Consumer surplus is the area below demand and above the price (area A). Its area can be calculated as

$$CS = \text{area } A = \frac{1}{2} \times \text{base} \times \text{height} = (0.5) \times (137{,}000 - 0) \times (\$7.60 - \$0.75)$$
$$= (0.5) \times 137{,}000 \times \$6.85 = \$469{,}225$$

Producer surplus is the area below price and above supply (area B):

$$PS = \text{area } B = \frac{1}{2} \times \text{base} \times \text{height} = (0.5) \times (137{,}000 - 0) \times (\$0.75 - \$0.02)$$
$$= 0.5 \times 137{,}000 \times \$0.73 = \$50{,}005$$



c. We can rearrange demand and supply to get inverse demand and supply, respectively:

$$Q^D = 152 - 20P$$
$$20P = 152 - Q^D$$
$$P = 7.6 - 0.05Q^D$$

and

$$Q^S = 188P - 4$$
$$188P = 4 + Q^S$$
$$P = \frac{4}{188} + \frac{Q^S}{188}$$

Using the integral equations for consumer and producer surplus, and substituting the inverse demand and supply equations and the equilibrium quantity and price as calculated in part (a), we can see that

$$CS = \int_0^{137}\big((7.6 - 0.05Q) - 0.75\big)dQ = \int_0^{137}(6.85 - 0.05Q)dQ$$
$$= \int_0^{137} 6.85\ dQ - \int_0^{137} 0.05Q\ dQ = [6.85Q]_0^{137} - \left[\frac{0.05Q^2}{2}\right]_0^{137}$$
$$= [6.85(137) - 6.85(0)] - \left[\frac{0.05(137)^2}{2} - \frac{0.05(0)^2}{2}\right]$$
$$= (938.45 - 0) - (469.225 - 0) = 469.225$$

Since quantity is measured in thousands, this is the same \$469,225 as found above.

$$PS = \int_0^{137}\left(0.75 - \left(\frac{4}{188} + \frac{Q}{188}\right)\right)dQ = \int_0^{137}\left(\frac{137}{188} - \frac{Q}{188}\right)dQ$$
$$= \int_0^{137} \frac{137}{188}\ dQ - \int_0^{137} \frac{Q}{188}\ dQ = \left[\frac{137}{188}Q\right]_0^{137} - \left[\frac{Q^2}{376}\right]_0^{137}$$
$$= \left[\frac{137}{188}(137) - \frac{137}{188}(0)\right] - \left[\frac{(137)^2}{376} - \frac{(0)^2}{376}\right]$$
$$= (99.8351 - 0) - (49.9176 - 0) = 49.9176$$

This is approximately the same \$50,005 identified in the problem with a difference due to rounding.