∂ 3.2 figure it out

A local tire market is represented by the following equations and in the diagram on the next page:

$$Q^D = 3{,}200 - 25P$$
$$Q^S = 15P - 800$$

where Q is the number of tires sold weekly and P is the price per tire. The equilibrium price is \$100 per tire, and 700 tires are sold each week.

Suppose an improvement in the technology of tire production makes them cheaper to produce so that sellers are willing to sell more tires at every price. Specifically, suppose that quantity supplied rises by 200 at each price.

a. What is the new supply curve?

b. What are the new equilibrium price and quantity?

c. What happens to consumer and producer surplus as a result of this change?

d. Calculate the old consumer and producer surplus using calculus.

e. Calculate the new consumer and producer surplus using calculus.

f. Using your answers from parts (d) and (e), show that the changes in consumer and producer surplus are the same (within rounding) as what you found in part (c).

Solution:

a. Quantity supplied rises by 200 units at every price, so we simply add 200 to the equation for Q^S:

$$Q_2^S = 15P - 800 + 200 = 15P - 600$$

b. The new equilibrium occurs where $Q^D = Q_2^S$:

$$3{,}200 - 25P = 15P - 600$$
$$3{,}800 = 40P$$
$$P = \$95$$

We can find the equilibrium quantity by substituting the equilibrium price into either the supply or demand equation (or both):

$$Q^D = 3{,}200 - 25(95) = 3{,}200 - 2{,}375 = 825 \qquad Q_2^S = 15(95) - 600 = 1{,}425 - 600 = 825$$

The new equilibrium quantity is 825 tires per week. Notice that because supply increased, the equilibrium price fell and the equilibrium quantity rose just as we would predict.

c. The easiest way to determine the changes in consumer and producer surplus is to use a graph such as the one on the next page. To calculate all of the areas involved, we need to make sure we calculate the demand choke price and the supply choke prices before and after the increase in supply.

The demand choke price is the price at which quantity demanded is zero:

$$Q^D = 0 = 3{,}200 - 25P$$
$$25P = 3{,}200$$
$$P = \$128$$

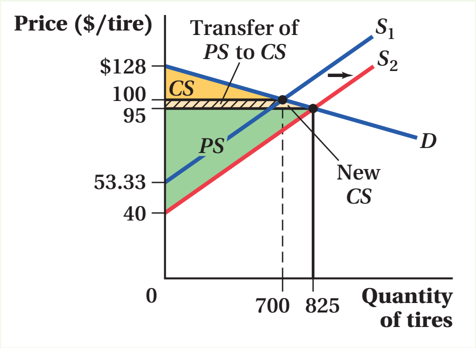
The demand choke price is \$128.

The supply choke price is the price at which quantity supplied is zero. Because supply is shifting, we need to calculate the supply choke price for each supply curve:

$$Q_1^S = 0 = 15P - 800$$
$$15P = 800$$
$$P = \$53.33$$
$$Q_2^S = 0 = 15P - 600$$
$$15P = 600$$
$$P = \$40$$

The initial supply choke price is \$53.33 but falls to \$40 when supply increases.

With the choke prices and the two equilibrium price and quantity combinations, we can draw the supply and demand diagram.



Consumer surplus: The initial consumer surplus is the area of the triangle below the demand curve but above the initial equilibrium price (\$100):

$$CS_{\text{initial}} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (700 - 0) \times (\$128 - \$100) = (0.5)(700)(\$28) = \$9{,}800$$

The new consumer surplus is the area of the triangle below the demand curve and above the new equilibrium price (\$95):

$$CS_{\text{new}} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (825 - 0) \times (\$128 - \$95) = (0.5)(825)(\$33) = \$13{,}612.50$$

So, after the outward shift in supply, consumer surplus rises by \$3,812.50.

Producer surplus: The initial producer surplus is the area of the triangle below the initial equilibrium price and above the initial supply curve (S_1):

$$PS_{\text{initial}} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (700 - 0) \times (\$100 - \$53.33) = (0.5)(700)(\$46.67) = \$16{,}334.50$$

The new producer surplus is the area of the triangle below the new equilibrium price and above the new supply curve (S_2):

$$PS_{\text{new}} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times (825 - 0) \times (\$95 - \$40) = (0.5)(825)(\$55) = \$22{,}687.50$$

The increase in supply also led to a rise in producer surplus by $6,353.

d. We can rearrange demand and supply to get inverse demand and supply, respectively:

$$Q^D = 3{,}200 - 25P$$
$$25P = 3{,}200 - Q^D$$
$$P = 128 - 0.04Q^D$$

and

$$Q^S = 15P - 800$$
$$15P = 800 + Q^S$$
$$P = \frac{800}{15} + \frac{Q^S}{15}$$

Using the integral equations for consumer and producer surplus, and substituting the inverse demand and supply equations and the initial equilibrium quantity and price as indicated in part (c), we can see that

$$CS = \int_0^{700}\big((128 - 0.04Q) - 100\big)dQ = \int_0^{700}(28 - 0.04Q)dQ$$
$$= \int_0^{700} 28\ dQ - \int_0^{700} 0.04Q\ dQ = [28Q]_0^{700} - \left[\frac{0.04Q^2}{2}\right]_0^{700}$$
$$= [28(700) - 28(0)] - \left[\frac{0.04(700)^2}{2} - \frac{0.04(0)^2}{2}\right]$$
$$= (19{,}600 - 0) - (9{,}800 - 0) = 9{,}800$$

This is the same $9,800 as found above.

$$PS = \int_0^{700} 100 - \left(\frac{800}{15} + \frac{Q}{15}\right)dQ = \int_0^{700}\left(\frac{700}{15} - \frac{Q}{15}\right)dQ$$
$$= \int_0^{700} \frac{700}{15}\ dQ - \int_0^{700} \frac{Q}{15}\ dQ = \left[\frac{700}{15}Q\right]_0^{700} - \left[\frac{Q^2}{30}\right]_0^{700}$$
$$= \left[\frac{700}{15}(700) - \frac{700}{15}(0)\right] - \left[\frac{(700)^2}{30} - \frac{(0)^2}{30}\right]$$
$$= (32{,}666.67 - 0) - (16{,}333.33 - 0) = 16{,}333.34$$

This is approximately the same $16,334.50 identified in the problem with a difference due to rounding.

e. The inverse form of the new supply curve is

$$Q^S = 15P - 600$$
$$15P = 600 + Q^S$$
$$P = 40 + \frac{Q^S}{15}$$

Using the integral equations for consumer and producer surplus, and substituting the inverse demand and supply equations and the new equilibrium quantity and price as indicated in part (b), we can see that

$$CS = \int_0^{825}\big((128 - 0.04Q) - 95\big)dQ = \int_0^{825}(33 - 0.04Q)dQ$$

$$= \int_0^{825} 33\ dQ - \int_0^{825} 0.04Q\ dQ = [33Q]_0^{825} - \left[\frac{0.04Q^2}{2}\right]_0^{825}$$

$$= [33(825) - 33(0)] - \left[\frac{0.04(825)^2}{2} - \frac{0.04(0)^2}{2}\right]$$

$$= (27{,}225 - 0) - (13{,}612.5 - 0) = 13{,}612.5$$

This is the same $13,612.50 found above.

$$PS = \int_0^{825}\left(95 - \left(40 + \frac{Q}{15}\right)\right)dQ = \int_0^{825}\left(55 - \frac{Q}{15}\right)dQ = \int_0^{825} 55\ dQ - \int_0^{825} \frac{Q}{15}\ dQ$$

$$= [55Q]_0^{825} - \left[\frac{Q^2}{30}\right]_0^{825}$$

$$= [55(825) - 55(0)] - \left[\frac{(825)^2}{30} - \frac{(0)^2}{30}\right]$$

$$= (45{,}375 - 0) - (22{,}687.5 - 0) = 22{,}687.5$$

This is the same $22,687.50 identified in the problem.

f. Since the initial and new consumer and producer surplus values are alike between the algebraic and calculus methods, the changes in these surplus measures also are alike.