

10. Jake and Paul run a paper company. Each week they need to produce 1,000 reams of paper to ship to their customers. The paper plant's long-run production function is  $Q = 4K^{0.75}L^{0.25}$ , where  $Q$  is the number of reams produced,  $K$  is the quantity of capital rented, and  $L$  is the quantity of labor hired. For this production function, the  $MP_L = K^{0.75}/L^{0.75}$  and the  $MP_K = 3L^{0.25}/K^{0.25}$ . The weekly cost function for the paper plant is  $C = 10K + 2L$ , where  $C$  is the total weekly cost.
- What ratio of capital to labor minimizes Jake and Paul's total costs?
  - How much capital and labor will Jake and Paul need to rent and hire in order to produce 1,000 reams of paper each week?
  - How much will hiring these inputs cost them?
  - Given the production function in this problem, show that the marginal products are as given using calculus.
  - Use a Lagrangian to solve the constrained cost-minimization problem for the producer.
  - Derive Jake and Paul's expansion path.
  - What is Jake and Paul's demand for capital?
  - Use calculus to confirm that the demand for capital satisfies the Law of Demand.