∂ 7.2 figure it out

Fields Forever is a small farm that grows strawberries to sell at the local farmers' market. It produces strawberries using 5 acres of land that it rents for $200 per week. Fields Forever also hires labor at a price of $250 per week per worker. The table below shows how the output of strawberries (measured in truckloads) varies with the number of workers hired:

LABOR (WORKERS PER WEEK)	QUANTITY OF STRAWBERRIES (TRUCKLOADS PER WEEK)
0	0
1	1
3	2
7	3
12	4
18	5

a. Calculate the marginal cost of 1 to 5 truckloads of strawberries for Fields Forever.

b. Suppose that Forever Fields faces a total cost function: $TC = Q^3 - Q^2 + Q - 100$. Derive the farm's marginal cost function using calculus.

c. For which values of Q is marginal cost as expressed in part (b) increasing?

Solution:

a. The easiest way to solve this problem is to add several columns to the table above. We should add fixed cost, variable cost, and total cost. Fixed cost is the cost of land that does not vary as output varies. Therefore, fixed cost is $200. Variable cost is the cost of labor. It can be found by multiplying the quantity of labor by the wage rate ($250). Total cost is the sum of fixed cost and variable cost.

LABOR PER WEEK	QUANTITY OF STRAWBERRIES (TRUCKLOADS)	FIXED COST, *FC*	VARIABLE COST, $VC = W \times L$	TOTAL COST, $TC = FC + VC$	MARGINAL COST, *MC*
0	0	$200	$250 × 0 = $0	$200	—
1	1	200	250 × 1 = 250	450	$250
3	2	200	250 × 3 = 750	950	500
7	3	200	250 × 7 = 1,750	1,950	1,000
12	4	200	250 × 12 = 3,000	3,200	1,250
18	5	200	250 × 18 = 4,500	4,700	1,500

Marginal cost is the change in total cost per unit increase in output, or $\Delta TC/\Delta Q$. When output rises from 0 units to 1 truckload of strawberries, total cost rises from $200 to $450. Therefore, the marginal cost of the first truckload of strawberries is $450 – $200 = $250. As output rises from 1 to 2 truckloads, total cost rises from $450 to $950, so the marginal cost is $950 – $450 = $500. When the third truckload is produced, total cost rises from $950 to $1,950 so marginal cost is $1,950 – $950 = $1,000. Production of the fourth truckload pushes total cost to $3,200, so the marginal cost of the fourth truckload is $3,200 – $1,950 = $1,250. When production rises from 4 to 5 truckloads, total cost rises from $3,200 to $4,700, so the marginal cost of the fifth truckload is $1,500.

We could have also calculated the marginal cost of each truckload by looking at only the change in variable cost (rather than the change in total cost). Because the amount of land is fixed, Fields Forever can only get more strawberries by hiring more labor and increasing its variable cost.

b. Marginal cost is the derivative of the total cost function with respect to Q.

Here,

$$\begin{aligned} MC &= \frac{dTC}{dQ} \\ &= 3Q^{3-1} - 2Q^{2-1} + Q^{1-1} - 0 \\ &= 3Q^2 - 2Q + 1 \end{aligned}$$

c. To find the quantities for which marginal cost is increasing, we need to find values for which the derivative of marginal cost with respect to Q is positive. Here,

$$\begin{aligned} \frac{dMC}{dQ} &= 3(2)Q^{2-1} - 2 \\ &= 6Q - 2 \end{aligned}$$

This is increasing when $6Q - 2 > 0$ or $Q > \frac{1}{3}$.