7.3 figure it out

Suppose a firm's total cost curve is $TC = 15Q^2 + 8Q + 45$, and $MC = 30Q + 8$.

a. Find the firm's fixed cost, variable cost, average total cost, and average variable cost.

b. Find the output level that minimizes average total cost.

c. Find the output level at which average variable cost is minimized.

d. Show that the firm's marginal cost is as given using calculus.

e. Minimize average total cost using calculus and confirm that your answer is the same as in part (b).

f. Minimize average variable cost using calculus and confirm that your answer is the same as in part (c).

Solution:

a. Fixed cost is a cost that does not vary as output changes. We can find FC by calculating total cost at zero units of output:

$$TC = 15(0)^2 + 8(0) + 45 = 45$$

Variable cost can be found by subtracting fixed cost from total cost:

$$VC = TC - FC = (15Q^2 + 8Q + 45) - 45 = 15Q^2 + 8Q$$

Notice that, as we have learned in the chapter, VC depends on output; as Q rises, VC rises.

Average total cost is total cost per unit or TC/Q:

$$ATC = \frac{TC}{Q} = \frac{15Q^2 + 8Q + 45}{Q}$$
$$= 15Q + 8 + \frac{45}{Q}$$

Average variable cost is variable cost per unit or VC/Q:

$$ATC = \frac{VC}{Q} = \frac{15Q^2 + 8Q}{Q}$$
$$= 15Q + 8$$

b. Minimum average total cost occurs when $ATC = MC$:

$$15Q + 8 + \frac{45}{Q} = 30Q + 8$$
$$15Q + \frac{45}{Q} = 30Q$$
$$\frac{45}{Q} = 15Q$$
$$15Q^2 = 45$$
$$Q^2 = 3$$
$$Q = \sqrt{3} = 1.732$$

c. Minimum average variable cost occurs where $AVC = MC$:

$$15Q + 8 = 30Q + 8$$
$$15Q = 0$$
$$Q = 0$$

d. Marginal cost is the derivative of the total cost function with respect to Q.

Here,

$$MC = \frac{dTC}{dQ} = 15(2)Q^{2-1} + 8 + 0 = 30Q + 8$$

e. Average total cost is shown in part (a): $ATC = 15Q + 8 + \frac{45}{Q} = 15Q + 8 + 45Q^{-1}$.

We can minimize this directly by calculating the first-order condition.

Taking the derivative of ATC with respect to Q, we see that

$$\frac{dATC}{dQ} = 15 + 0 - 45Q^{-2}$$
$$= 15 - 45Q^{-2}$$

Setting this equal to zero, $15 - 45Q^{-2} = 0$ or $Q^2 = 3$ or $Q = 1.732$.

To confirm that this is a minimum (and not a maximum), we can check the second-order condition:

$$\frac{d^2ATC}{dQ^2} = 0 - 2(-45)Q^{-2-1} = 90Q^{-3} > 0$$

Since this is positive, we know that we have a minimum. This is the same answer as found in part (b).

f. Average variable cost is shown in part (a): $AVC = 15Q + 8$. We can minimize this directly by calculating the first-order condition.

Taking the derivative of AVC with respect to Q, we see that $\frac{dAVC}{dQ} = 15 > 0$, which indicates that AVC is increasing in Q. Since this does not depend on Q, the function is minimized at $Q = 0$. This is the same answer as found in part (c).