7.4 figure it out

Steve and Sons Solar Panels has a production function represented by $Q = 4KL$, where the $MP_L = 4K$ and the $MP_K = 4L$. The current wage rate (W) is \$8 per hour, and the rental rate on capital (R) is \$10 per hour.

a. In the short run, the plant's capital stock is fixed at $K = 10$. What is the cost the firm faces if it wants to produce $Q = 200$ solar panels?

b. What will the firm wish to do in the long run to minimize the cost of producing $Q = 200$ solar panels? How much will the firm save? (*Hint*: You may have to review Chapter 6, to remember how a firm optimizes when both labor and capital are flexible.)

c. Show that the marginal products are as given using calculus.

Solution:

a. If capital is fixed at $K = 10$ units, then the amount of labor needed to produce $Q = 200$ units of output is

$$Q = 4KL$$
$$200 = 4(10)L = 40L$$
$$L = 5$$

Steve and Sons would have to hire 5 units of labor. Total cost would be

$$TC = WL + RK = \$8(5) + \$10(10) = \$40 + \$100 = \$140$$

b. In Chapter 6, we learned that in the long run, a firm minimizes costs when it produces a quantity at which the marginal rate of technical substitution of labor for capital equals the ratio of the costs of labor (wage) and capital (rental rate): $MRTS_{LK} = W/R$. We know that

$$MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{4K}{4L} = \frac{K}{L}$$
$$\frac{W}{R} = \frac{8}{10}$$

To minimize costs, the firm will set $MRTS_{LK} = W/R$:

$$10K = 8L$$
$$K = 0.8L$$

To produce $Q = 200$ units, we can substitute for K in the production function and solve for L:

$$Q = 200 = 4KL = 4(0.8L)(L)$$
$$200 = 3.2L^2$$
$$L^2 = 62.5$$
$$L = 7.91$$
$$K = 0.8L = (0.8)(7.91) = 6.33$$

To minimize cost, the firm will want to increase labor from 5 to 7.91 units and reduce capital from 10 to 6.33 units. Total cost will fall to

$$TC = WL + RK = \$8(7.91) + \$10(6.33) = \$63.28 + \$63.30 = \$126.58$$

Therefore, the firm will save \$140 – \$126.58 = \$13.42.

c. The marginal product of labor is the partial derivative of the production function with respect to labor, and the marginal product of capital is the partial derivative of the production function with respect to capital. Here, $MP_L = \frac{\partial Q}{\partial L} = 4K$ and $MP_K = \frac{\partial Q}{\partial K} = 4L$. This is as given.