## ∂ 7.5 figure it out

Suppose that the long-run total cost function for a firm is $LTC = 22{,}600Q - 300Q^2 + Q^3$ and its long-run marginal cost function is $LMC = 22{,}600 - 600Q + 3Q^2$.

a. At what levels of output will the firm face economies of scale? Diseconomies of scale? (*Hint*: These cost functions yield a typical U-shaped long-run average cost curve.)

b. Minimize long-run average total cost using calculus and confirm that your answer is the same as in part (a).

**Solution:**

a. If we can find the output that minimizes long-run average total cost, we can determine the output levels for which the firm faces economies and diseconomies of scale. We know that when $LMC < LATC$, long-run average total cost is falling and the firm experiences economies of scale. Likewise, when $LMC > LATC$, the long-run average total cost curve slopes up and the firm faces diseconomies of scale. So, if we can figure out where the minimum $LATC$ occurs, we can see where economies of scale end and diseconomies begin.

Minimum average cost occurs when $LMC = LATC$. But, we need to determine $LATC$ before we begin. Long-run average total cost is long-run total cost divided by output:

$$LATC = \frac{LTC}{Q} = \frac{22{,}600Q - 300Q^2 + Q^3}{Q}$$
$$= 22{,}600 - 300Q + Q^2$$

Now, we need to set $LATC = LMC$ to find the quantity that minimizes $LATC$:

$$LATC = LMC$$
$$22{,}600 - 300Q + Q^2 = 22{,}600 - 600Q + 3Q^2$$
$$300Q = 2Q^2$$
$$300 = 2Q$$
$$Q = 150$$

Long-run average total cost is minimized and economies of scale are constant when the firm produces 150 units of output. Thus, at $Q < 150$, the firm faces economies of scale. At $Q > 150$, the firm faces diseconomies of scale. (You can prove this to yourself by substituting different quantities into the long-run average total cost equation and seeing if $LATC$ rises or falls as $Q$ changes.)

b. Long-run average total cost is shown in part (a): $LATC = 22{,}600 - 300Q + Q^2$. We can minimize this directly by calculating the first-order condition.

Taking the derivative of $LATC$ with respect to $Q$, we see that

$$\frac{dLATC}{dQ} = 0 - 300 + 2Q^{2-1}$$
$$= 2Q - 300$$

Setting this equal to zero, $2Q - 300 = 0$ or $Q = 150$. This is the same answer as found in part (a).