## ∂ 9.2 figure it out

Babe's Bats (BB) sells baseball bats for children around the world. The firm faces a demand curve of $Q = 10 - 0.4P$, where $Q$, is measured in thousands of baseball bats, and $P$ in dollars per bat. BB has a marginal cost curve that is equal to $MC = 5Q$.

a. Solve for BB's profit-maximizing level of output. Show the firm's profit-maximization decision graphically.

b. What price will BB charge to maximize its profit?

c. Assuming $TC = 2.5Q^2$, solve for BB's profit-maximizing level of output and price using calculus.

d. Confirm that BB is maximizing and not minimizing its profit function.

### Solution:

a. To solve this problem, we should follow the three-step procedure outlined in the text. First, we need to derive the marginal revenue curve for BB bats. Because the firm faces a linear demand curve, the easiest way to obtain the marginal revenue curve is to start by solving for the firm's inverse demand curve:

$$Q = 10 - 0.4P$$
$$0.4P = 10 - Q$$
$$P = 25 - 2.5Q$$

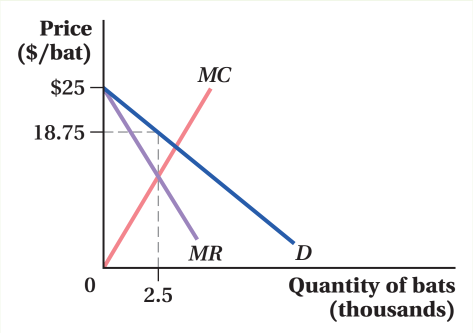
For this inverse demand curve, $a = 25$ and $b = 2.5$. Therefore, since $MR = a - 2bQ$, we know that BB's $MR$ curve will be

$$MR = 25 - 2(2.5Q) = 25 - 5Q$$

To solve for the profit-maximizing level of output, we can follow the profit-maximization rule $MR = MC$:

$$MR = MC$$
$$25 - 5Q = 5Q$$
$$10Q = 25$$
$$Q^* = 2.5$$

Therefore, BB should produce 2,500 bats. This profit-maximization decision is shown in the figure at the top of next column. Profit is maximized at the output level at which the marginal revenue and marginal cost curves intersect.




b. Now we need to plug $Q^*$ from (a) into the inverse demand curve to obtain the profit-maximizing price:

$$P^* = 25 - 2.5Q = 25 - 2.5(2.5) = \$18.75 \text{ per bat}$$

Babe's Bats is maximizing its profit when it sells 2,500 baseball bats at a price of $18.75 per bat. A shortcut to solving is to begin with the profit-maximizing condition $MR = MC$, as we did in part (a). In general, beginning with the profit-maximizing condition is easiest for firms with linear demand curves and simple cost functions. However, some firms have more complicated demand curves and total cost functions. For these firms, solving the profit-maximization problem directly using calculus may save you some work.

c. First, we need to set up Babe's profit-maximization problem:

$$\max_{Q} \pi = TR(Q) - TC(Q) = PQ - TC(Q)$$

Because Babe's Bats has some market power, its choice of $Q$ affects the price. So, we need to use the demand curve for BB's bats to solve for price as a function of quantity, or the firm's inverse demand curve:

$$Q = 10 - 0.4P$$
$$0.4P = 10 - Q$$
$$P = 25 - 2.5Q$$

Substituting this expression for $P$ and the total cost curve into the profit function, we find

$$\pi = TR - TC = PQ - TC$$
$$= (25 - 2.5Q)Q - 2.5Q^2$$
$$= 25Q - 2.5Q^2 - 2.5Q^2 = 25Q - 5Q^2$$

So, the firm's profit-maximization problem is

$$\max_Q \pi = 25Q - 5Q^2$$

The first-order condition for this problem is

$$\frac{d\pi}{dQ} = \frac{d(25Q - 5Q^2)}{dQ} = 0$$

$$25 - 10Q = 0$$

$$10Q = 25$$

$$Q^* = 2.5 \text{ or } 2{,}500 \text{ bats}$$

d. We need to check the second-order condition for the profit-maximization problem in order to show this. In Figure It Out 9A.1, we saw that $d\pi/dQ = 25 - 10Q$. The second derivative of the profit function therefore is

$$\frac{d^2\pi}{dQ^2} = -10$$

Since this is less than zero, we have confirmed that BB has maximized, not minimized, its profit function.