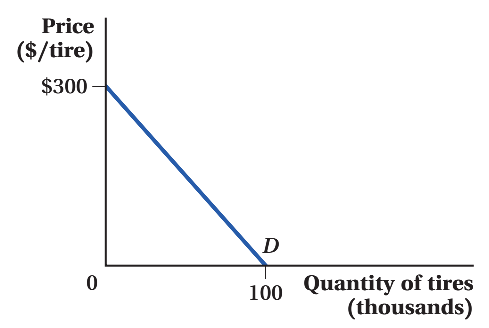
∂ 9.3 figure it out

The Power Tires Company has market power and faces the demand curve shown in the figure below. The firm's marginal cost curve is $MC = 30 + 3Q$.



a. What is the firm's profit-maximizing output and price?

b. If the firm's demand changes to $P = 240 - 2Q$ while its marginal cost curve remains the same, what is the firm's profit-maximizing level of output and price? How does this compare to your answer for (a)?

c. Draw a diagram showing these two outcomes. Holding marginal cost equal, how does the shape of the demand curve affect the firm's ability to charge a high price?

d. For the original demand, calculate the marginal revenue curve using calculus and confirm that it is the same as derived within the answer to part (a).

e. Suppose that total cost for this problem is $TC = 30Q + 1.5Q^2$. Set up Power Tires Company's profit-maximization problem using the original demand scenario and solve for the company's profit-maximizing level of output and price using calculus.

f. For the new demand in part (b), calculate the marginal revenue curve using calculus and confirm that it is the same as derived within the answer to that part of the question.

g. Again supposing that total cost for this problem is $TC = 30Q + 1.5Q^2$, set up Power Tires Company's profit-maximization problem using the new demand scenario and solve for the company's profit-maximizing level of output and price using calculus.

Solution:

a. To solve for the firm's profit-maximizing level of output, we need to find the firm's marginal revenue curve. But, we only have a diagram of the demand curve. So, we will start by solving for the inverse demand function. The inverse demand function will typically have the form

$$P = a - bQ$$

where a is the vertical intercept and b is the absolute value of the slope $\left(= \left|\frac{\Delta P}{\Delta Q}\right|\right)$. We can see from the figure of the demand curve that $a = 300$. In addition, we can calculate the absolute value of the slope of the demand curve as $\left|\frac{\Delta P}{\Delta Q}\right| = \left|\frac{-300}{100}\right| = 3$. Therefore, $b = 3$. This means that the demand for Power Tires is

$$P = 300 - 3Q$$

We know that the equation for marginal revenue (when demand is linear) is $P = a - 2bQ$. Therefore,

$$MR = 300 - 6Q$$

Setting marginal revenue equal to marginal cost, we find

$$MR = MC$$
$$300 - 6Q = 30 + 3Q$$
$$270 = 9Q$$
$$Q = 30$$

To find price, we substitute $Q = 30$ into the firm's demand equation:

$$P = 300 - 3Q$$
$$= 300 - 3(30) = 210$$

The firm should produce 30,000 tires and sell them at a price of $210.

b. If demand changes to $P = 240 - 2Q$, marginal revenue becomes $MR = 240 - 4Q$ because now $a = 240$ and $b = 2$. Setting $MR = MC$, we find

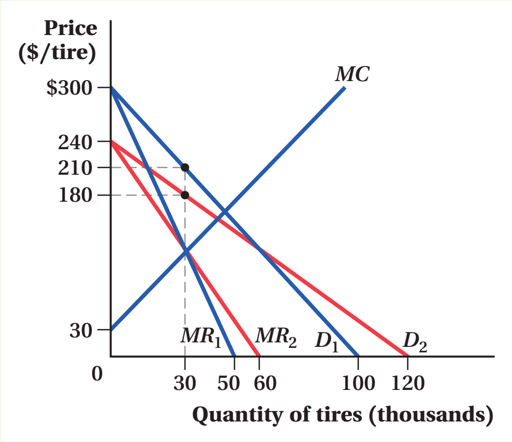
$$240 - 4Q = 30 + 3Q$$
$$210 = 7Q$$
$$Q = 30$$

Even with reduced demand, the firm should still produce 30 units if it wants to maximize profit. Substituting into the new demand curve, we can see that the price will be

$$P = 240 - 2Q$$
$$= 240 - 2(30) = 180$$

Here, the equilibrium price is lower even though the profit-maximizing output is the same.

c. The new diagram appears below. Because D_2 is flatter than D_1, the firm must charge a lower price. Consumers are more responsive to price.



d. The revenue function is

$$TR = P \times Q = (300 - 3Q)Q = 300Q - 3Q^2$$
$$MR = dR/dQ = 300 - 6Q$$

This is the same as the answer to part (a).

e. Power Tires Company's profit-maximization problem is

$$\max_Q \pi = TR(Q) - TC(Q) = PQ - TC(Q) = (300 - 3Q)Q - (30Q + 1.5Q^2) = 270Q - 4.5Q^2$$

Taking the first-order condition gives

$$\frac{d\pi(Q)}{dQ} = \frac{d(270Q - 4.5Q^2)}{dQ} = 270 - 9Q = 0$$
$$Q = 30$$

At this quantity, price is $P = 300 - 3(30) = 210$, as in part (a).

f. The revenue function now is

$$TR = P \times Q = (240 - 2Q)Q = 240Q - 2Q^2$$
$$MR = \frac{dR}{dQ} = 240 - 4Q$$

This is the same as the answer to part (b).

g. Power Tires Company's profit-maximization problem is now

$$\max_Q \pi = TR(Q) - TC(Q) = PQ - TC(Q) = (240 - 2Q)Q - (30Q + 1.5Q^2) = 210Q - 3.5Q^2$$

Taking the first-order condition gives

$$\frac{d\pi(Q)}{dQ} = \frac{d(210Q - 3.5Q^2)}{dQ} = 210 - 7Q = 0$$
$$Q = 30$$

At this quantity, price is $P = 240 - 2(30) = 180$, as in part (b).