$\partial 9.4$ figure it out

Let's return to our earlier problem regarding Babe's Bats (BB). Remember that BB faces an inverse demand curve of $P=25-2.5 Q$ and a marginal cost curve $M C=5 Q$.
a. Calculate the deadweight loss from market power at the firm's profit-maximizing level of output.
b. Calculate the deadweight loss from market power at the firm's profit-maximizing level of output using calculus and confirm that this is the same answer as what is determined using geometry in part (a).

## Solution:

a. The easiest way to find deadweight loss is to use a diagram. Therefore, we should start by drawing a graph with demand, marginal revenue, and marginal cost:

We know from our earlier problem that the profit-maximizing level of output is 2,500 bats sold at a price of $\$ 18.75$.

To find the deadweight loss from market power, we need to consider the con-
 sumer and producer surplus and compare it with the competitive outcome. If BB participated in a competitive market, it would set price equal to marginal cost to determine its output:

$$
\begin{aligned}
P & =M C \\
25-2.5 Q & =5 Q \\
25 & =7.5 Q \\
Q & =3.33
\end{aligned}
$$

Therefore, BB would sell 3,333 bats. Of course, the price will be lower at this level of output:

$$
\begin{aligned}
P & =25-2.5 Q \\
& =25-2.5(3.33) \\
& =16.68
\end{aligned}
$$

If the market were competitive, the bats would sell for $\$ 16.68$ each. Consumer surplus would be areas $A+B+C$ (the area below the demand curve and above the competitive price), and producer surplus would be areas $D+E+F$ (the area below the competitive price but above the marginal cost curve). Total surplus would be areas $A+B+C+D+$ $E+F$.

When BB exercises its market power, it reduces its output to 2,500 bats and increases its price to $\$ 18.75$. In this situation, consumer surplus is only area $A$ (the area below demand but above the monopoly price). Producer surplus is areas $B+D+F$ (the area below the monopoly price but above marginal cost). Total surplus under market power is $A+B+D+F$.

So, what happens to areas $C$ and $E$ ? Area $C$ was consumer surplus but no longer exists. Area $E$ was producer surplus but also has disappeared. These areas are the deadweight loss from market power. We can calculate this area by measuring the area of the triangle that encompasses areas $C+E$. To do so, we have one more important calculation to make. We need to be able to calculate the height of the triangle, so we need to determine the marginal cost of producing 2,500 units:

$$
\begin{aligned}
M C & =5 Q \\
& =5(2.5) \\
& =12.5
\end{aligned}
$$

Now, we can calculate the area of the deadweight loss triangle:

$$
\begin{aligned}
D W L & =\text { Areas } C+E=\frac{1}{2} \times \text { Base } \times \text { Height } \\
& =\frac{1}{2} \times(3.33-2.5) \times(\$ 18.75-\$ 12.50) \\
& =\frac{1}{2} \times 0.83 \times \$ 6.25 \\
& =\$ 2.59375
\end{aligned}
$$

Remember that the quantity is measured in thousands, so the deadweight loss is equal to $\$ 2,593.75$.
b. Deadweight loss can be calculated using integration, as in the online appendix to Chapter 3. Here,

$$
\begin{aligned}
D W L & =\int_{2.5}^{3.33}((25-2.5 Q)-(5 Q)) d Q=\int_{2.5}^{3.33}(25-7.5 Q) d Q \\
& =\int_{2.5}^{3.33} 25 d Q-\int_{2.5}^{3.33} 7.5 Q d Q \\
& =[25 Q]_{2.5}^{3.33}-\left[\frac{7.5 Q^{2}}{2}\right]_{2.5}^{3.33} \\
& =[25(3.33)-25(2.5)]-\left[\frac{7.5(3.33)^{2}}{2}-\frac{7.5(2.5)^{2}}{2}\right] \\
& =[(83.25-62.5)-(41.5834-23.4375)=20.75-18.1459 \approx 2.60
\end{aligned}
$$

This corresponds to deadweight loss on the order of $\$ 2,600$, as in part (a), with minor differences due to rounding across the two methods.

