Solution

10. a. The inverse demand curve is

$$Q = 40 - 0.5P$$
$$P = 80 - 2Q$$

The marginal revenue is

$$MR = 80 - 4Q$$

b.

$$MR = 80 - 4Q = 10 = MC$$
$$4Q = 70$$
$$Q = 17.5$$

The profit-maximizing level of output is 17.5 tons of bentonite.

c. The profit-maximizing price is

$$17.5 = 40 - 0.5P$$
$$0.5P = 40 - 17.5$$
$$P = \$45$$

d. If the marginal cost is $20 + Q$, the profit-maximizing quantity is

$$MR = 80 - 4Q = 20 + Q = MC$$
$$5Q = 60$$
$$Q = 12$$

The profit-maximizing price is

$$12 = 40 - 0.5P$$
$$P = \$56$$

e. We first need the revenue function. To get this, we can find the inverse form of the demand curve and plug it into revenue for P:

$$Q = 40 - 0.5P$$
$$0.5P = 40 - Q$$
$$P = 80 - 2Q$$
$$TR = PQ = (80 - 2Q)Q = 80Q - 2Q^2$$

Then,

$$MR = \frac{dMR}{dQ} = 80 - 4Q$$

f. The monopolist's profit-maximization problem is

$$\max_Q \pi = TR - TC = (80Q - 2Q^2) - (10Q) = 70Q - 2Q^2$$

Solving,

$$\frac{d\pi}{dQ} = 70 - 4Q = 0$$
$$70 = 4Q$$
$$Q = 17.5$$

Subbing into the demand equation to derive price, we get

$$P = 80 - 2Q = 80 - 2(17.5) = 45$$

g. The monopolist's marginal cost is $\frac{dTC}{dQ} = 20 + Q$.

h. The monopolist's profit-maximization problem is

$$\max_{Q} \pi = TR - TC = (80Q - 2Q^2) - (20Q - 0.5Q^2) = 60Q - 2.5Q^2$$

Solving,

$$\frac{d\pi}{dQ} = 60 - 5Q = 0$$

$$60 = 5Q$$

$$Q = 12$$

Subbing into the demand equation to derive price, we get

$$P = 80 - 2Q = 80 - 2(12) = 56$$