Chapter 2 Online Appendix: The Calculus of Equilibrium and Elasticities

This text provides a number of math supplements—from the chapter Figure It Out features to the in-text calculus appendices and math review appendix. Why add another appendix? These online appendices give us an opportunity to illustrate additional—and often more advanced—mathematical techniques that can be applied to the economics you learn throughout the course. Some online appendices will build on concepts explored in the chapters themselves, whereas others will provide richer detail on the mathematics in the in-text calculus appendices. As with the other math supplements, the online appendices provide information and techniques that may further your understanding of economics, but are not intended as substitutes for what you would learn in your college mathematics courses.

In this first online appendix, we use calculus to describe the effect of changes in variables other than the good's own price on equilibrium outcomes. We also calculate and apply a variety of supply and demand elasticities using calculus, enabling us to calculate elasticities in a wider variety of circumstances than those shown in the text.

Demand

A **demand curve** is a representation of a relationship between the quantity of a good that consumers demand and that good's price, holding all other factors constant. Let's consider the hypothetical linear demand curve for tomatoes from the text: $Q^D = 1{,}000 - 200P$, where Q^D is the quantity demanded of tomatoes in pounds and P is the price in dollars per pound. Note that this demand equation is written so that quantity is a function of price only.

The definition of a demand curve, however, is more general. Particularly, it specifies that demand may also depend on "other factors" (which we then hold constant for our calculations of demand curves). In the chapter we learned that when other factors change (such as changes in income, the prices of related goods, or tastes), the demand curve shifts. It is therefore useful to think about an "expanded" demand function for which quantity demanded is a function not only of the good's own price but also of some of these other factors.

As a relatively simple example, consider adding the prices of related goods and income to the linear demand curve: $Q^D = 900 - 200P + 100P_s - 600P_c + 0.01I$, where P_s and P_c are the prices of a substitute good and of a complementary good respectively and I is income. Why might we include the prices of substitute and complementary goods in the demand curve for tomatoes? As we saw in the text, a change in the price of a substitute for a good (a good that can be used in place of another good) affects demand for the original good. Similarly, a change in the price of a good that is used in combination with another (a complement) also affects the good's demand.

Let's use the demand for tomatoes to explore these relationships more fully. Suppose peppers are a substitute for tomatoes with price P_s, and the people enjoy eating lettuce with tomatoes, so it is a complementary good with price P_c.

Assume lettuce and peppers each cost \$1 per unit, and that the average consumer income is \$60,000. Note that at these values $Q^D = 900 - 200P + 100(1) - 600(1) + 0.01(60{,}000)$. Simplifying this, we see that the demand curve written only as a function of price is $Q^D = 1{,}000 - 200P$—the same relationship as in the text!

What happens if the price of peppers—the substitute—increases to \$2? An increase in the price of a substitute product increases the quantity demanded of the original good at all prices, all else equal, thereby shifting the original demand curve to the right. We can see this by plugging the new price of peppers into the original demand curve: $Q^D = 900 - 200P + 100(2) - 600(1) + 0.01(60{,}000) = 1{,}100 - 200P$. The slope[1] of the demand curve is the same: $-1/200$. Because the intercept[2] has increased from 5 to 5.5, the price increase has resulted in a parallel shift up and to the right of the demand curve.

Now let's add another layer of changes to the demand curve. Suppose that the price of lettuce increases to \$2. The quantity demanded of tomatoes becomes $Q^D = 900 - 200P + 100(2) - 600(2) + 0.01(60{,}000) = 500 - 200P$. In this example, the price increases of peppers and lettuce are equivalent—both the substitute and complement experienced price increases from \$1 to \$2. You might then expect that these two price increases would cause demand shifts of equivalent magnitudes. Instead, given this particular demand function, the effect (shift of the demand curve down and to the left) of the increase in the price of lettuce (a complement to tomatoes) more than offsets the magnitude of the rightward shift due to the increased price of peppers (the substitute good). As a result, the new demand curve lies to the left of the original demand curve. We can use partial derivatives to show differences in the magnitudes of the two effects and to draw further conclusions.

Because the demand curve is now multivariable (quantity demanded depends not just on own price, but many other variables as well), it is useful to consider what the partial derivatives reviewed in the book's math review appendix tell us in this context. As a starting point, consider the partial derivative of the demand curve with respect to its own price $\frac{\partial Q^D}{\partial P}$. This partial derivative isolates the effect of own price P on quantity demanded Q^D. It can be interpreted similarly to $\frac{\Delta Q^D}{\Delta P}$ studied in the text. The partial derivative holds all the other factors in our multivariable setting constant.

Next, consider the partial derivative of the demand curve with respect to the price of the substitute good $\frac{\partial Q^D}{\partial P_s}$. This partial derivative isolates the effect of the substitute's price P_s on quantity demanded Q^D, holding everything else constant. Likewise, the partial derivative of the demand curve with respect to the complement good's price $\frac{\partial Q^D}{\partial P_c}$ isolates the effect of P_c on Q^D, holding everything else constant. Finally, $\frac{\partial Q^D}{\partial I}$ isolates the effect of income on quantity demanded, holding everything else constant.

We can use the partial derivatives of quantity demanded with respect to each variable to demonstrate a series of economic concepts from Chapter 2. Going back to our example, $Q^D = 900 - 200P + 100P_s - 600P_c + 0.01I$, note that

$$\frac{\partial Q^D}{\partial P} = -200 < 0 \tag{1}$$

$$\frac{\partial Q^D}{\partial P_s} = 100 > 0 \tag{2}$$

$$\frac{\partial Q^D}{\partial P_c} = -600 < 0 \tag{3}$$

$$\frac{\partial Q^D}{\partial I} = 0.01 > 0 \tag{4}$$

[1] Recall that since quantity is on the x-axis and price is on the y-axis, the slope of the line is $1/(\Delta Q/\Delta P)$.

[2] The intercept is from the inverse form of the demand curve with price expressed as a funtion of quantity demanded.

What do these partial derivatives tell us about the demand for this product (tomatoes)? First, most demand curves are downward-sloping by the law of demand, which states that as price increases, quantity demanded decreases, all else equal. This is precisely what the negative relationship in (1) tells us.

Partial derivatives with respect to the prices of other goods can tell us whether a good is a substitute (2) or a complement (3). Recall that a substitute is a good for which a price increase translates into a quantity increase in the market for the original good and a complement is a good for which there is a negative relationship between the price in one market and the quantity demanded in another. Expressed mathematically in terms of calculus, this means that $\frac{\partial Q^D}{\partial P_s} > 0$ and $\frac{\partial Q^D}{\partial P_c} < 0$, exactly as shown in (2) and (3).

The partial derivative with respect to income (4) can reveal whether we are dealing with a normal or an inferior good. Recall that a normal good is a good for which quantity demanded rises when income rises and an inferior good is a good for which quantity demanded decreases when income rises. In terms of calculus, this means that $\frac{\partial Q^D}{\partial I} > 0$ for a normal good and $\frac{\partial Q^D}{\partial I} < 0$ for an inferior good.[3] Because we find a positive relationship between income and quantity demanded for our tomato example in (4), we know that tomatoes are normal goods in this particular case.

Supply

A **supply curve** illustrates the relationship between the quantity supplied of a good and its price, holding all other factors constant. The supply curve we examine for tomatoes is $Q^S = 200P - 200$. As in the demand case, this equation is written with quantity as a function of price alone, although changes in other factors can cause the supply curve to shift. Here, consider an "expanded" supply function where quantity supplied is a function not only of a good's own price but also of suppliers' costs of production. As an example, let's consider the case of tomato seeds and fertilizer, two inputs to the production of tomatoes. Now, quantity supplied can be written $Q^S = 200P - 100P_1 - 5P_2 - 50$, where P_1 is the price of input 1 and P_2 is the price of input 2. In particular, let's set P_1 to be the price of tomato seeds at \$1 per pound, and P_2 as the price of fertilizer at \$10 per bag. Substituting into the supply relationship, we see that $Q^S = 200P - 100(1) - 5(10) - 50 = 200P - 200$. Note that this yields the same function for the supply curve as in the chapter.

We can analyze the effects of changes in input prices in a way that is similar to how we analyzed the effects on demand of changes in the prices of other goods and in income. Particularly, suppose that the price of fertilizer decreases to \$5. Now, $Q^S = 200P - 100(1) - 5(5) - 50 = 200P - 175$. When the price of an input falls, production becomes cheaper. The quantity supplied increases at every price, and the supply curve shifts out to the right. Note that in this example (like in the examples for demand shifts in the previous section), the change described is the result of a supply curve *shift*, as opposed to a movement along a given supply curve. When the price of fertilizer decreases, therefore, $Q^S = 200P - 175$ instead of $Q^S = 200P - 200$. The supply curve has shifted because suppliers are now willing to produce more at every price.

Just as they are for demand, partial derivatives are a useful tool for further examining the effects of price changes on the supply of a good. Consider the partial derivative

[3] In Chapter 5, you will see a special (and very rare) case of inferior goods called **Giffen goods,** which have the characteristic that own price and quantity demanded are positively related. In the language of partial derivatives, this means that for Giffen goods $\frac{\partial Q^D}{\partial P} > 0$ while $\frac{\partial Q^D}{\partial I} < 0$ (because Giffen goods are a type of inferior goods).

of the supply curve with respect to a good's own price $\frac{\partial Q^S}{\partial P}$. This partial derivative now isolates the effect of own price P on quantity supplied Q^S and can be interpreted as the change in quantity supplied for a given change in price, holding all other factors in this multivariable setting constant. In our example, $\frac{\partial Q^S}{\partial P} = 200 > 0$. The fact that this partial derivative is positive is consistent with the law of supply, which indicates that as price increases, quantity supplied increases so that supply curves generally have a positive slope.

In addition to the partial derivative of the quantity supplied with respect to own price, consider the partial derivatives of the supply curve with respect to the prices of the inputs $\frac{\partial Q^S}{\partial P_1}$ and $\frac{\partial Q^S}{\partial P_2}$. These partial derivatives isolate the effects of the input prices on quantity supplied Q^S. We expect that quantity supplied will decrease if input prices rise (and will increase if input prices fall), all else equal. Therefore, our expectation is that $\frac{\partial Q^S}{\partial P_1} < 0$ and $\frac{\partial Q^S}{\partial P_2} < 0$. For our example, we find that this is true: $\frac{\partial Q^S}{\partial P_1} = -100 < 0$ and $\frac{\partial Q^S}{\partial P_2} = -5 < 0$.

Comparative Statics

By extending our supply and demand framework to one of many variables, we can now model the effects on equilibrium price and quantity of a change in variables other than a good's own price. This type of before and after modeling of an equilibrium is often called *comparative statics*. As a starting point, consider the equilibrium condition (quantity demanded equals quantity supplied) with variables in addition to the good's own price included. We can then solve for a new equilibrium after a change in the value of an additional variable, and solve for the new equilibrium using the algebraic methods in Chapter 2. An example is provided in the following Figure It Out.

2OA.1 figure it out

Let's continue the example from Figure It Out 2.3 in the textbook. As in the text, suppose that the supply of lemonade is represented by $Q^S = 40P$, where quantity is measured in pints and price is measured in cents per pint.

a. Now suppose that the demand for lemonade is $Q^D = 7{,}000 - 10P - 0.02I$, where I is income. What are the current equilibrium price and quantity if income is \$100,000?

b. Suppose that income increases to \$125,000. What is the new equation for the demand of lemonade?

c. What will be the new equilibrium price and quantity of lemonade after the income increase?

d. Is lemonade a normal or an inferior good? Answer the question using a partial derivative and its interpretation.

Solution:

a. The original equilibrium price and quantity are found by substituting income into the demand relationship and then setting this equal to the supply side:

$$\begin{aligned} Q^D &= 7{,}000 - 10P - 0.02(100{,}000) \\ &= 7{,}000 - 10P - 2{,}000 \\ &= 5{,}000 - 10P \end{aligned}$$

Note that the demand side is equivalent to the demand curve, as given in the problem in the text.

Setting quantity demanded equal to quantity supplied, we get

$$\begin{aligned} Q^D &= Q^S \\ 5{,}000 - 10P &= 40P \\ 50P &= 5{,}000 \\ P &= 100 \end{aligned}$$

This corresponds to \$1 since price is in cents.

To obtain the quantity, we can substitute into the supply or demand equations (or both to check):

$$Q^D = 5{,}000 - 10(100) = 4{,}000 \text{ or } Q^S = 40(100) = 4{,}000$$

This corresponds to 4,000 pints.

b. Now, $Q^D = 7{,}000 - 10P - 0.02(125{,}000) = 4{,}500 - 10P$. Note that the increase in income decreases quantity demanded at each price and results in a parallel shift of the demand curve downward and to the left.

c. Setting quantity demanded equal to quantity supplied yields

$$\begin{aligned} Q^D &= Q^S \\ 4{,}500 - 10P &= 40P \\ 50P &= 4{,}500 \\ P &= 90 \end{aligned}$$

To get quantity, we can substitute P into the supply or demand equation (or both to check):

$$Q^D = 4{,}500 - 10(90) = 3{,}600 \text{ or } Q^S = 40(90) = 3{,}600$$

Thus, price is now \$0.90 and quantity is 3,600 pints.

Note that as income increases, both quantity and price fall.

d. Lemonade is an inferior good here because $\frac{\partial Q^D}{\partial I} = -0.02 < 0$ and therefore quantity demanded decreases with income. This is consistent with the directions of the quantity and price changes identified above.

Figure 2.11 in the text examines how an equal magnitude shift of the demand curve (and supply curve) affects equilibrium price and quantity differently depending on the relative steepness or flatness of the supply curve (and demand curve). Note that for the example in the Figure It Out above, the supply curve is relatively flat. Consider instead a supply curve of $Q^S = 10P + 3{,}000$, which is steeper. Note that despite having a different slope and intercept, this curve has the property of going through the original equilibrium because

$$\begin{aligned}
Q^D &= Q^S \\
5{,}000 - 10P &= 10P + 3{,}000 \\
20P &= 2{,}000 \\
P &= 100 \\
Q^D &= 5{,}000 - 10(100) = 4{,}000 \\
Q^S &= 10(100) + 3{,}000 = 4{,}000
\end{aligned}$$

Now consider the effects of an increase in income to \$125,000. Setting the new demand curve as in the Figure It Out example equal to this alternate supply, we find that $4{,}500 - 10P = 10P + 3{,}000$ or $P = 75$. To obtain the quantity, we can substitute into the supply or demand equations (or both to check). From demand, $Q^D = 4{,}500 - 10(75) = 3{,}750$. The same income increase therefore leads to a lower equilibrium price and higher equilibrium quantity ($P = 75$, $Q = 3{,}750$) when supply is steeper, and a higher equilibrium price and lower quantity ($P = 90$, $Q = 3{,}600$) when supply is flatter. Thus, the price decrease is greater in the case of steep supply, and the quantity decrease is greater in the case of flat supply. This is the same pattern identified in the discussion around Figure 2.11. (Note, however, that the figure illustrates a rightward shift of demand, as opposed to the leftward one in this problem.) The methods here can be used to extend beyond the case of just one curve shifting, to cases in which both supply and demand simultaneously shift, resulting in an even wider range of possible outcomes.

We can also use calculus to examine comparative statics for marginal changes in variables other than price. As an example, let's look at the case of a change in income. The equilibrium condition expressed as a function of income is $Q^D(P(I),I) = Q^S(P(I))$. We want to know how price and quantity change with income, but we don't want to hold everything else constant in the background (i.e., we want to analyze the effects of an income change on actual equilibrium price and quantity). To start, we can differentiate the equilibrium with respect to income[4]:

$$\frac{\partial Q^D}{\partial P}\frac{dP}{dI} + \frac{\partial Q^D}{\partial I} = \frac{\partial Q^S}{\partial P}\frac{dP}{dI}$$

Rearranging this equation, we can see that

$$\begin{aligned}
\frac{\partial Q^D}{\partial I} &= \frac{\partial Q^S}{\partial P}\frac{dP}{dI} - \frac{\partial Q^D}{\partial P}\frac{dP}{dI} \\
&= \frac{dP}{dI}\left(\frac{\partial Q^S}{\partial P} - \frac{\partial Q^D}{\partial P}\right)
\end{aligned}$$

We can then solve for the derivative of price with respect to income:

$$\frac{dP}{dI} = \frac{\dfrac{\partial Q^D}{\partial I}}{\dfrac{\partial Q^S}{\partial P} - \dfrac{\partial Q^D}{\partial P}}$$

[4] This condition uses the chain rule from calculus. This rule states that for the function $f(x) = g(h(x))$, the derivative can be calculated as $\frac{df(x)}{dx} = \frac{dg(x)}{dh(x)} \times \frac{dh(x)}{dx}$.

To make this more concrete, let's reconsider the case in the previous Figure It Out in which $Q^D = 7{,}000 - 10P - 0.02I$ and $Q^S = 40P$. For that example,

$$\frac{dP}{dI} = \frac{-0.02}{40 - (-10)} = -0.0004$$

This means that the \$25,000 increase in income in that example lowers the equilibrium price by \$10 ($-0.0004 \times \$25{,}000$). Likewise, the change in quantity is $\frac{dQ^S}{dI} = \frac{\partial Q^S}{\partial P}\frac{dP}{dI}$ $= 40(-0.0004) = -0.016$, so the \$25,000 increase in income lowers equilibrium quantity by 400 ($-0.016 \times \$25{,}000$). These are the same changes in the price and quantity of lemonade found in the Figure It Out example.

Elasticities

Note that the effect of an income change on equilibrium quantity is different from the effect of an income change on quantity demanded (or likewise on quantity supplied). For the latter, we can use calculus to calculate a partial derivative, as we did in part (d) of the Figure It Out exercise. The partial derivative $\frac{\partial Q^D}{\partial I}$, for example, lets us know that for a very small increase in income—holding all else equal—quantity demanded changes by $\frac{\partial Q^D}{\partial I}$ units. These units, however, are difficult to interpret. As a result, economists turn to another calculation that allows for easier interpretation: elasticities.

In calculus terms, elasticities are partial derivatives. Consider the price elasticity of demand, which can be written as

$$E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D}$$

where Q^D is quantity demanded and P is price.

Note the similarities to the formula at the top of page 47 of the text. The price elasticity of supply can be written as $E^S = \frac{\partial Q^S}{\partial P}\frac{P}{Q^S}$. Similarly, the income elasticity of demand can be written as $E_I^D = \frac{\partial Q^D}{\partial I}\frac{I}{Q^D}$, where I is income, and the cross-price elasticity of demand can be written using subscripts as in the chapter as $E_{XY}^D = \frac{\partial Q_X^D}{\partial P_Y}\frac{P_Y}{Q_X^D}$, where P_Y is the price of a related good (e.g., a substitute or a complement) to good X.

Total Expenditure and the Price Elasticity of Demand

We can also use calculus to derive the relationship between (1) total expenditure changes from price changes and (2) the elasticity of demand. Total expenditure, and likewise total revenue, can be expressed as $R(P) = P \times Q^D(P)$, where quantity is expressed as a function of price. We can therefore think about maximizing this function with respect to P:

$$\max_P \; P \times Q^D(P)$$

We can find the first-order condition by taking the derivative of the total expenditure function with respect to P and then setting this derivative equal to zero. The first-order condition[5] therefore is

$$\frac{dQ^D(P)}{dP}P + Q^D(P) = 0$$

[5] This first-order condition uses the product rule from calculus. This rule states that for the function $f(x) = g(x) \times h(x)$, the derivative can be calculated as $\frac{df(x)}{dx} = \frac{dh(x)}{dx}g(x) + \frac{dg(x)}{dx}h(x)$.

Rearranging, we can see that

$$\frac{dQ^D(P)}{dP}P = -Q^D(P)$$

$$\frac{dQ^D(P)}{dP}\frac{P}{Q^D(P)} = -1$$

Note that the left side of this equality is simply the price elasticity of demand[6], so $E^D = -1$. This means that expenditure is maximized when demand is unit-elastic.[7] When demand is inelastic (when the price elasticity of demand is less than 1 in absolute value), consumer expenditure increases with a price increase. Expressed with calculus, this means that for inelastic demand, $\frac{dR(P)}{dP} > 0$. This can be shown by examining the case that $\frac{dQ^D(P)}{dP}P + Q^D(P) > 0$. Rearranging this, $\frac{dQ^D(P)}{dP}P > -Q^D(P)$. Dividing both sides by $Q^D(P)$, we see that $E^D > -1$. Because E^D is a negative number for a downward-sloping demand curve, this condition corresponds to inelastic demand.

Similarly, for elastic demand (when the price elasticity of demand is greater than 1 in absolute value), consumer expenditure decreases for a price increase, meaning that $\frac{dR(P)}{dP} < 0$. Therefore, only in the case of unit-elastic demand (when the price elasticity of demand is exactly 1 in absolute value) would consumer expenditure be at a maximum (as it should be given the optimization exercise outlined above)!

2OA.2 figure it out

Suppose that the inverse demand curve for hospital scrubs in bright floral prints can be expressed as $P = 80 - 10Q^{0.5}$, where price is in dollars and quantity is in thousand sets (top and bottom). What is the price elasticity of demand at a quantity of 25,000 sets?

Solution:

Note that this is a nonlinear demand curve since quantity Q is raised to a power other than 1 and therefore the relationship between price and quantity is nonlinear. As a tool, calculus provides a way to calculate the price elasticity of demand for this equation at a given quantity. First, note that when quantity is 25,000 sets, $P = 80 - 10(25)^{0.5} = 80 - 10(5) = 30$, and therefore a set of floral scrubs costs \$30. The price elasticity of demand formula is $E^D = \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} = \frac{1}{\frac{\partial P}{\partial Q^D}}\frac{P}{Q^D}$. Using the demand curve equation, we can see that $\frac{\partial P}{\partial Q^D} = -10(0.5)Q^{0.5-1} = -5Q^{-0.5}$. At a quantity of 25,000 sets, this value is $-5(25)^{-0.5} = -1$. Making the appropriate substitutions, we find that $E^D = \frac{1}{-1}\frac{30}{25} = -1.2$. Sets of floral hospital scrubs therefore are price-elastic, because hospital workers may switch to other patterns when price increases. Notice that this method can be used similarly for income and cross-price elasticities in situations when demand is nonlinear and for supply elasticities when supply is nonlinear.

[6] The price elasticity of demand here is expressed as a standard derivative instead of a partial derivative since we wrote the total expenditure function to be a function of just P.

[7] See Figure 2.19 in the text for a graphical representation. Formal optimization techniques (for finding maxima and minima) and a review of derivatives are presented in the Math Review Appendix in the back of your textbook.

Problems

1. Suppose that the supply of specialty workstation laptops is represented by $Q^S = 1{,}000 + P$, where price is measured in dollars and quantity is measured in units.
   a. Now suppose that the demand for the laptops is $Q^D = 9{,}000 - P - 0.05I$, where I is income. What are the current equilibrium price and quantity if income is \$100,000?
   b. Suppose that income falls to \$80,000. What is the new equation for the demand?
   c. What will be the new equilibrium price and quantity after the income decrease?
   d. Is the laptop workstation a normal or an inferior good? Answer the question using a partial derivative.
2. Suppose that the supply of a flat panel TV stand is represented by $Q^S = 8P - 20P_i - 200$, where P is the price of the stand and P_i is the price of the hardware needed to hold the stand together. All prices are in dollars and quantity is in units. Assume that the current hardware price is \$5.
   a. Suppose that the demand for the TV stand is $Q^D = 4{,}700 - 2P + 0.5I$, where P is the price and I is a representative household's income. What are the current equilibrium price and quantity if income is \$1,000?
   b. Suppose that income falls to \$800. What is the new equation for the demand for TV stands as a function of price P? Does this correspond to an increase or decrease in the demand for the TV stands? Does the demand curve shift to the left or right?
   c. Suppose that the price of the hardware increases to \$6. What is the new equation for the supply of TV stands as a function of price P? Does this correspond to an increase or decrease in supply? Does the supply curve shift to the left or right?
   d. What will be the new equilibrium price and quantity of the TV stand after the changes in supply and demand [after all changes in parts (b) and (c)]?
3. From the original setup in Problem 2, suppose that the quantity supplied of flat panel TV stands is represented by $Q^S = 8P - 20P_i - 200$, where P is the price of the stand and P_i is the price of hardware inputs, and that quantity demanded is $Q^D = 4{,}700 - 2P + 0.5I$, where I is income. Assume that at the equilibrium, income is \$1,000 and the hardware price is \$5.
   a. Calculate the income elasticity of demand using calculus.
   b. Calculate the input elasticity of supply using calculus. (*Hint*: Think of cross-price elasticities on the demand side as being analogous to "input" elasticities on the supply side.)
4. Suppose that the inverse demand curve for a dinner-for-two special at a small local restaurant can be expressed as $P = 4{,}900 - 3Q^2$, where price is expressed in dollars and quantity in number of specials. What is the price elasticity of demand when 40 specials are purchased?