Chapter 10 Online Appendix:
The Calculus of Price Strategies

Chapter 10 of the textbook explores the market structures that tend to give rise to price discrimination and the optimal pricing strategies of firms in each of these market structures. In Chapter 9, we demonstrated that there was a deadweight loss associated with market power. Keep in mind that the market power in Chapter 9 was for a single-price monopolist (i.e., the monopolist charged every buyer the same price). In this appendix, we show that in the special case of a perfect price-discriminating monopolist, there is no deadweight loss associated with market power.

In the final section, we demonstrate that a firm with market power can use a three-tiered pricing strategy to capture producer surplus that is greater than the producer surplus enjoyed by a single-price monopolist.

Perfect Price-Discriminating Monopolist and Producer Surplus: A Numerical Example

Suppose that the demand for high-end customized stereos in a particular city is given by $q_D = 200 - 0.01P$, where P is the price for a customized stereo. There is only one firm in this market providing such stereos. To simplify the analysis, we assume that there are no fixed costs. With no fixed costs, the firm's total variable cost and total cost are equivalent. The firm's total cost of providing high-end customized stereos is given by $TC(q_{ppdm}) = \frac{1}{30}q_{ppdm}^3 - 5q_{ppdm}^2 + 1{,}250q_{ppdm}$, where q_{ppdm} is the output choice of the perfect price-discriminating monopolist. Notice that in this example

$$TC(q_{ppdm}) = \int_0^{q_{ppdm}} MC(q)dq$$

where $MC(q)$ is the marginal cost function.

The perfect price-discriminating monopolist can charge a different price to each customer. Because we assume the firm in this market structure knows the value of the good to each customer, the firm can set each customer's price to be her marginal benefit for the customized stereo. Marginal benefit is given by the inverse demand function. Using the demand function above, we have $MB(q) = P = 20{,}000 - 100q$. This pricing strategy implies that the firm's total revenue is

$$TR(q_{ppdm}) = \int_0^{q_{ppdm}} MB(q)dq = \int_0^{q_{ppdm}} (20{,}000 - 100q)dq$$

Evaluating the definite integral, we identify this perfect price-discriminating monopolist's total revenue function as

$$\int_0^{q_{ppdm}} (20{,}000 - 100q)dq = (20{,}000q - 50q^2)\big|_0^{q_{ppdm}}$$

$$TR(q_{ppdm}) = 20{,}000q_{ppdm} - 50q_{ppdm}^2$$

The firm's objective is to choose q_{ppdm} to maximize profits. In general, profits are

$$\pi = TR(q_{ppdm}) - TC(q_{ppdm})$$

The output level that maximizes firm profit is the same output level that maximizes producer surplus. In this example, profit and producer surplus are exactly the same (this is because we have assumed that there are no fixed costs for this firm). Formally, producer surplus is

$$PS = TR(q_{ppdm}) - \int_0^{q_{ppdm}} MC(q)dq$$

Using our earlier result for total cost and the definite integral of marginal cost over our range of output, we find that $PS = TR(q_{ppdm}) - TC(q_{ppdm})$, which is identical to our statement of firm profit. Given the total revenue function we derived and the total cost function, the profit function is

$$\pi = 20{,}000q_{ppdm} - 50q_{ppdm}^2 - \frac{1}{30}q_{ppdm}^3 + 5q_{ppdm}^2 - 1{,}250q_{ppdm}$$

Combining like terms allows us to simplify profit as

$$\pi = 18{,}750q_{ppdm} - 45q_{ppdm}^2 - \frac{1}{30}q_{ppdm}^3$$

The firm's first-order condition for maximization is

$$0 = \frac{d\pi}{dq_{ppdm}} = 18{,}750 - 90q_{ppdm} - \frac{1}{10}q_{ppdm}^2$$

Because this equation is in the form of a quadratic equation, we may apply the quadratic formula to identify the choice of q_{ppdm} that satisfies our first-order condition:

$$q_{ppdm} = \frac{90 \pm \sqrt{8{,}100 + 4\left(\frac{1}{10}\right)18{,}750}}{-\frac{1}{5}}$$

$$= (-1{,}074.5,\ 174.5)$$

For a maximization, we require $\frac{d^2\pi}{dq_{ppdm}^2} < 0$. This implies

$$\frac{d^2\pi}{dq_{ppdm}^2} = -90 - \frac{1}{5}q_{ppdm} < 0$$

$$q_{ppdm} > -450$$

Our profit-maximizing choice of output for the perfect price-discriminating monopolist must be 174.5. Profit and producer surplus at this quantity is $PS = 18{,}750(174.5) - 45(174.5)^2 - \frac{1}{30}(174.5)^3 = 1{,}724{,}494.80$.

Perfect Price-Discriminating Monopolist and Consumer Surplus With the optimal choice of the firm identified, let's now consider consumer surplus. Consumer surplus is given by

$$CS = \int_0^{q_m} \big(MB(q) - P(q)\big)dq$$

where q_m is the market output level. Because the firm sets the price at each increment of the output so $P(q) = MB(q)$, we see that

$$CS = \int_0^{q_m} \big(MB(q) - MB(q)\big)dq = \int_0^{q_m} 0dq = 0$$

Notice that this result will hold for all perfect price-discriminating market settings.

Perfect Price-Discriminating Monopolist and Total Surplus For a market outcome to be efficient, total surplus TS (the sum of producer surplus and consumer surplus) must be maximized. Producer surplus is defined as

$$PS = \int_0^{q_m} \big(P(q) - MC(q)\big)dq$$

With this expression for producer surplus and the expression we identified for consumer surplus in the previous section, we can write total surplus as

$$TS = \int_0^{q_m} \big(MB(q) - P(q)\big)dq + \int_0^{q_m} \big(P(q) - MC(q)\big)dq$$

We can simplify this definite integral as

$$TS = \int_0^{q_m} \big(MB(q) - MC(q)\big)dq$$

Given that a perfect price-discriminating monopolist chooses $P(q) = MB(q)$, producer surplus is exactly the full amount of total surplus in this market structure. That is,

$$TS = \int_0^{q_m} \big(MB(q) - MC(q)\big)dq + \int_0^{q_m} \big(P(q) - MC(q)\big)dq = PS$$

Thus, the firm's choice of output to maximize profits corresponds precisely with the condition for efficiency in this market setting. If the firm accomplishes the profit-maximizing objective, the market outcome is efficient.

10OA.1 figure it out

Suppose that a firm with market power can perfectly price-discriminate and faces the demand function $q = 400 - 4P$. The firm's total production costs are given by $MC(q) = 2q + 0.12q^2$.

a. If the firm cannot price-discriminate, identify the firm's profit-maximizing output and price.

b. If the firm cannot price-discriminate, identify consumer surplus and producer surplus in the market, assuming that the firm maximizes profits. Calculate the deadweight loss from market power.

c. If the firm can perfectly price-discriminate, identify the firm's output level.

d. If the firm can perfectly price-discriminate, identify consumer surplus, producer surplus, and the deadweight loss from market power.

Solution:

a. Using the firm's demand function, we can find the inverse demand function:

$$P = 100 - 0.25q$$

This implies that total revenue is given by

$$TR(q) = P(q) \times q = 100q - 0.25q^2$$

Marginal revenue is therefore

$$MR \equiv \frac{dTR(q)}{dq} = 100 - 0.5q$$

To maximize profit, the non-price-discriminating firm must produce the quantity at which $MR = MC$:

$$MR = 100 - 0.5q = 2q + 0.12q^2 = MC$$

Solving for q requires us to identify q such that $-0.12q^2 - 2.5q + 100 = 0$. Using the quadratic formula, we find that

$$q = \frac{2.5 \pm \sqrt{6.25 + 4(0.12)100}}{-0.24}$$
$$= (-41.11, 20.27)$$

The second-order condition for profit maximization requires

$$\frac{d^2\pi}{dq^2} = -0.24q - 2.5 < 0$$
$$q > -10.42$$

Thus, we conclude that the profit-maximizing level of output is $q = 20.27$. The optimal price is therefore $P = 94.9325$.

b. For a non-price-discriminating firm, producer surplus is

$$\begin{aligned} PS &= \int_0^{20.27} (94.9325 - 2q - 0.12q^2)dq \\ &= (94.9325q - q^2 - 0.04q^3)\big|_0^{20.27} \\ &= 94.9325(20.27) - (20.27)^2 - 0.04(20.27)^3 \\ &= 1{,}180.27 \end{aligned}$$

Consumer surplus is

$$\begin{aligned} CS &= \int_0^{20.27} (100 - 0.25q - 94.9325)dq \\ &= \int_0^{20.27} (5.0675 - 0.25q)dq \\ &= (5.0675q - 0.125q^2)\big|_0^{20.27} \\ &= 5.0675(20.27) - 0.125(20.27)^2 \\ &= 51.36 \end{aligned}$$

The maximum possible total surplus in the market with a nondiscriminating monopolist is 1,231.63. Next, we're going to determine the DWL associated with production of the good by a single-price monopolist. To do so, we must first calculate the maximum total surplus associated with this market:

$$\begin{aligned} TS &= \int_0^{q_{TS}} \big(MB(q) - MC(q)\big)dq \\ &= \int_0^{q_{TS}} (100 - 0.25q - 2q - 0.12q^2)dq \\ &= \int_0^{q_{TS}} (100 - 2.25q - 0.12q^2)dq \\ &= 100q - 1.125q^2 - 0.04q^3\big|_0^{q_{TS}} \\ &= 100q_{TS} - 1.125q_{TS}^2 - 0.04q_{TS}^3 \end{aligned}$$

The value of q_{TS} that maximizes TS occurs where the following first-order condition is satisfied:

$$0 = \frac{dTS}{dq_{TS}} = 100 - 2.25q_{TS} - 0.12q_{TS}^2$$

Using the quadratic formula, we identify

$$\begin{aligned} q &= \frac{2.25 \pm \sqrt{5.0625 + 4(0.12)100}}{-0.24} \\ &= (-39.73,\ 20.98) \end{aligned}$$

The second-order sufficient condition for total surplus maximization requires

$$\frac{d^2TS}{dq_{TS}^2} = -2.25 - 0.24q_{TS} < 0$$
$$q_{TS} > -9.375$$

Total surplus is maximized when $q_{TS} = 20.98$. Total surplus at this output level is

$$\begin{aligned} TS &= 100(20.98) - 1.125(20.98)^2 - 0.04(20.98)^3 \\ &= 1{,}233.44 \end{aligned}$$

Subtracting total surplus that accrues from market power (1,231.63) from the maximum possible total surplus (1,233.44), we find that the deadweight loss from market power is 1.81.

c. When the firm can engage in price discrimination by varying price across consumers, the general statement of producer surplus is

$$PS = \int_0^{q_{ppdm}} \big(P(q) - MC(q)\big)dq$$

The perfect price-discriminating monopolist sets $P(q) = MB(q) = 100 - 0.25q$ and $MC(q) = 2q + 0.12q^2$. Substituting these functions into the general statement of producer surplus yields

$$\begin{aligned} PS &= \int_0^{q_{ppdm}} (100 - 0.25q - 2q - 0.12q^2)dq \\ &= \int_0^{q_{ppdm}} (100 - 2.25q - 0.12q^2)dq \end{aligned}$$

Notice that maximization of this statement for PS is equivalent to the maximization of TS identified in part (b). Thus, we see that the perfect price-discriminating monopolist will choose the same output level as the choice that maximizes total surplus. That is, $q_{ppdm} = 20.98$.

d. Our work in part (c) demonstrates that producer surplus for a perfect price-discriminating monopolist is 1,233.44. Consumer surplus is

$$CS = \int_0^{20.98} \big(MB(q) - P(q)\big)dq$$

Because the perfect price-discriminating monopolist sets $P(q) = MB(q)$, we have

$$\begin{aligned} CS &= \int_0^{20.98} \big(MB(q) - MB(q)\big)dq \\ &= 0 \end{aligned}$$

Total surplus is the sum of producer surplus and consumer surplus. This is 1,233.44 for the perfect price-discriminating monopolist. In part (b), we demonstrated that total surplus is as large as possible at the value 1,233.44. Hence, the deadweight loss from market power is zero in this market structure.

Block Pricing

A firm engages in block pricing when it sets its price for a range of output the customer purchases. To entice the consumer to purchase more, the firm offers price breaks, provided it is profitable, for output in a higher range. To understand why block pricing enhances producer surplus for the firm, consider the following example for a screen-printing firm. The firm offers organizations an opportunity to have their organization logo printed onto sweatshirts. The demand for these sweatshirts is given by $q = 160 - 2P$, where P is the price per sweatshirt. The marginal cost of producing a sweatshirt for an organization is constant at $MC = 10$. The firm also has zero fixed costs (this implies that total variable costs and total costs are equivalent).

Identifying Revenues from a Three-Tiered Block Pricing Strategy Let's suppose that the firm wishes to offer sweatshirts for sale with three pricing tiers. We assume that the demand information and cost structure defined earlier are the same. That is, demand is $q = 160 - 2P$ and $MC = 10$. Let's identify the three-tiered pricing structure that maximizes the firm's producer surplus. Notice that for the first (highest-priced) tier, the number of units purchased (q_1) is $q_1 = 160 - 2p_1$. First-tier revenue is

$$\begin{aligned} TR_1 &= (160 - 2p_1)p_1 \\ &= 160p_1 - 2p_1^2 \end{aligned}$$

The number of units purchased in the second tier is $q_2 = 160 - q_1 - 2p_2$. That is, to identify the number of units purchased in the second tier, we must identify the excess demand for Tier 2 units after Tier 1 purchases are subtracted from Tier 1 demand. Inserting Tier 1 demand and simplifying, Tier 2 demand is $q_2 = 2p_1 - 2p_2$. Second-tier revenue is

$$\begin{aligned} TR_2 &= (2p_1 - 2p_2)p_2 \\ &= 2p_1p_2 - 2p_2^2 \end{aligned}$$

Finally, the number of units purchased in the third tier is $q_3 = 160 - q_1 - q_2 - 2p_3$. We identify the number of units purchased in Tier 3 by subtracting the number of units purchased in each of the first and second pricing tiers. That is, $q_3 = 160 - (160 - 2p_1) - (2p_1 - 2p_2) - 2p_3$. Simplifying, third-tier units purchased are determined according to $q_3 = 2p_2 - 2p_3$. Third-tier revenue is

$$\begin{aligned} TR_3 &= (2p_2 - 2p_3)p_3 \\ &= 2p_2p_3 - 2p_3^2 \end{aligned}$$

Total revenue aggregated across all price tiers is

$$TR = 160p_1 - 2p_1^2 + 2p_1p_2 - 2p_2^2 + 2p_2p_3 - 2p_3^2$$

Identifying Total Costs in a Three-Tiered Block Pricing Strategy With total revenue identified, we now turn to the task of identifying the firm's production cost associated with providing the output. This total production cost is

$$\begin{aligned} TC &= \int_0^{160-2p_1} MC(q)dq + \int_{160-2p_1}^{160-2p_1+2p_1-2p_2} MC(q)dq + \int_{160-2p_1+2p_1-2p_2}^{160-2p_1+2p_1-2p_2+2p_2-2p_3} MC(q)dq \\ &= \int_0^{160-2p_1} MC(q)dq + \int_{160-2p_1}^{160-2p_2} MC(q)dq + \int_{160-2p_2}^{160-2p_3} MC(q)dq \end{aligned}$$

Recall that marginal cost is constant at \$10 per unit for all output. Making this substitution and evaluating the definite integral yield

$$TC = (1{,}600 - 20p_1) + (1{,}600 - 20p_2 - 1{,}600 + 20p_1) + (1{,}600 - 20p_2 - 1{,}600 + 20p_3)$$

$$TC = 1{,}600 - 20p_3$$

Deriving the Optimal Three-Tiered Block Pricing Strategy With the firm's total revenue and total cost functions identified, we may express producer surplus as

$$PS = 160p_1 - 2p_1^2 + 2p_1p_2 - 2p_2^2 + 2p_2p_3 - 2p_3^2 - 1{,}600 + 20p_3$$

The firm's objective is to choose p_1, p_2, and p_3 such that PS is maximized. This maximization problem generates the system of first-order conditions:

$$\begin{aligned} 0 &= \frac{\partial PS}{\partial p_1} = 160 - 4p_1 + 2p_2 \\ 0 &= \frac{\partial PS}{\partial p_2} = 2p_1 - 4p_2 + 2p_3 \\ 0 &= \frac{\partial PS}{\partial p_3} = 2p_2 - 4p_3 + 20 \end{aligned}$$

From the first first-order condition, we have $p_1 = 40 + 0.5p_2$. From the third first-order condition, we have $p_3 = 5 + 0.5p_2$. Next, if we manipulate the second first-order condition, we have

$$p_2 = 0.5p_1 + 0.5p_3$$

Substituting, we identify

$$\begin{aligned} p_2 &= 0.5(40 + 0.5p_2) + 0.5(5 + 0.5p_2) \\ p_2 &= 20 + 0.25p_2 + 2.5 + 0.25p_2 \\ 0.5p_2 &= 22.5 \\ p_2 &= 45 \end{aligned}$$

With the Tier 2 price identified, we recognize that the optimal choice for the Tier 1 price is \$62.50 and the optimal choice for the Tier 3 price is \$27.50. This pricing structure implies that $q_1 = 160 - 2(62.50) = 35$, $q_2 = 160 - q_1 - 2p_2 = 160 - 35 - 2(45) = 35$, and $q_3 = 160 - q_1 - q_2 - 2p_3 = 160 - 35 - 35 - 2(27.50) = 35$. That is, $q_1 = 35$, $q_2 = 35$, and $q_3 = 35$.

The firm offers the first 35 sweatshirts for \$62.50 per sweatshirt. The second set of 35 sweatshirts is sold for \$45 per sweatshirt. Finally, for all sweatshirts ordered above 70 units, the firm charges \$27.50 per sweatshirt. This three-tiered block pricing structure generates revenue of \$4,725 from each client organization with production costs of \$1,050. The firm's producer surplus is \$3,675.

The Single-Price Monopolist's Producer Surplus To demonstrate that this three-tiered block pricing structure is superior to a single-price strategy for the firm, let's identify the optimal choice of a single-price monopoly. The firm's inverse demand function is $P = 80 - 0.5q$. This means that the firm's total revenue function is $TR = 80q - 0.5q^2$. The single-price monopolist's producer surplus is

$$\begin{aligned} PS &= 80q - 0.5q^2 - 10q \\ &= 70q - 0.5q^2 \end{aligned}$$

The single-price monopolist's objective is to choose q such that PS is maximized. The corresponding first-order condition is

$$0 = \frac{dPS}{dq} = 70 - q$$

Solving the first-order condition identifies that the single-price producer surplus-maximizing choice of output is 70 sweatshirts. The single price at which the monopolist may charge and sell 70 sweatshirts is \$45. The single-price monopolist generates revenues of \$3,150. Producing 70 sweatshirts costs \$700. Thus, the single-price monopolist's producer surplus is \$2,450. We see that the firm is able to capture an additional \$1,125 in producer surplus by switching from a single-price pricing structure to a three-tiered block pricing structure.

10OA.2 figure it out

A firm with market power faces the demand function $q = 2{,}000 - 40P$. The firm's marginal cost function is $MC(q) = 10 + 0.002q$.

a. If the firm behaves as a single-price monopoly, identify the firm's optimal price and output level.

b. If the firm behaves as a single-price monopoly, identify consumer surplus, producer surplus, and total surplus.

c. If the firm establishes a block pricing structure with two different prices, identify the two prices the firm will use to maximize producer surplus.

d. If the firm establishes a block pricing structure with two different prices and maximizes producer surplus, identify consumer surplus, producer surplus, and total surplus.

Solution:

a. The firm's inverse demand function is $P = 50 - 0.025q$. The total revenue function is $TR(q) = 50q - 0.025q^2$. Taking the first derivative of $TR(q)$ with respect to q produces the marginal revenue function

$$MR(q) = 50 - 0.05q$$

Profit maximization requires choosing the output level where $MR(q) = MC(q)$. For this market, we have

$$MR(q) = 50 - 0.05q = 10 + 0.002q = MC(q)$$
$$q = 769.23$$

At this output level, the price of the output is \$30.77.

b. For the single-price monopolist, producer surplus is

$$\begin{aligned} PS &= \int_0^{769.23} (30.77 - 10 - 0.002q)dq \\ &= \int_0^{769.23} (20.77 - 0.002q)dq \\ &= (20.77q - 0.001q^2)\Big|_0^{769.23} \\ &= 15{,}385.19 \end{aligned}$$

Consumer surplus is

$$\begin{aligned} CS &= \int_0^{769.23} (50 - 0.025q - 30.77)dq \\ &= \int_0^{769.23} (19.23 - 0.025q)dq \\ &= (19.23q - 0.0125q^2)\Big|_0^{769.23} \\ &= 7{,}395.86 \end{aligned}$$

Total surplus in the single-price monopolist market is \$22,781.05.

c. For a block pricing structure with two different prices, we will first decompose total revenue from each price tier. From the statement of the inverse demand function, we may identify $q_1 = 2{,}000 - 40p_1$ at the tier 1 price p_1. Total revenue from the high-price tier is

$$TR_1 = 2{,}000p_1 - 40p_1^2$$

Excess demand for output at the lower-price tier is

$$\begin{aligned} q_2 &= 2{,}000 - 40p_2 - q_1(p_1) \\ &= 2{,}000 - 40p_2 - 2{,}000 + 40p_1 \\ &= 2{,}000 - 2{,}000 + 40p_1 - 40p_2 \\ &= 40p_1 - 40p_2 \end{aligned}$$

Total revenue from this lower-price tier is

$$TR_2 = 40p_1p_2 - 40p_2^2$$

The firm's combined total revenue function is

$$TR = 2{,}000p_1 - 40p_1^2 + 40p_1p_2 - 40p_2^2$$

The firm's total production cost for output associated with Tier 1 pricing is

$$\begin{aligned} TC_1 &= \int_0^{2{,}000-40p_1}(10 + 0.002q)dq \\ &= (10q + 0.001q^2)|_0^{2{,}000-40p_1} \\ &= 10(2{,}000 - 40p_1) + 0.001(2{,}000 - 40p_1)^2 \\ &= 20{,}000 - 400p_1 + 4{,}000 - 160p_1 + 1.6p_1^2 \\ TC_1 &= 24{,}000 - 560p_1 + 1.6p_1^2 \end{aligned}$$

The firm's total production cost for output associated with Tier 2 pricing is

$$\begin{aligned} TC_2 &= \int_{2{,}000-40p_1}^{2{,}000-40p_1+40p_1-40p_2}(10 + 0.002q)dq \\ &= (10q + 0.001q^2)|_{2{,}000-40p_1}^{2{,}000-40p_2} \\ &= 10(2{,}000 - 40p_2) + 0.001(2{,}000 - 40p_2)^2 - 10(2{,}000 - 40p_1) - 0.001(2{,}000 - 40p_1)^2 \\ TC_2 &= 24{,}000 - 560p_2 + 1.6p_2^2 - 24{,}000 + 560p_1 - 1.6p_1^2 \end{aligned}$$

The firm's total production cost for producing output across both pricing blocks is

$$TC = 24{,}000 - 560p_2 + 1.6p_2^2$$

The block pricing monopolist's producer surplus is

$$PS = 2{,}000p_1 - 40p_1^2 + 40p_1p_2 - 40p_2^2 - 24{,}000 + 560p_2 - 1.6p_2^2$$

$$PS = 2{,}000p_1 - 40p_1^2 + 40p_1p_2 + 560p_2 - 41.6p_2^2 - 24{,}000$$

The firm's objective is to choose p_1 and p_2 that maximize PS. The relevant system of first-order conditions is

$$0 = \frac{\partial PS}{\partial p_1} = 2{,}000 - 80p_1 + 40p_2$$

$$0 = \frac{\partial PS}{\partial p_2} = 40p_1 + 560 - 83.2p_2$$

Using the first first-order condition, we may solve for p_1. This produces

$$p_1 = 25 + 0.5p_2$$

Inserting the right-hand side of this statement for p_1 into the second first-order condition, we have

$$\begin{aligned} 0 &= 40(25 + 0.5p_2) + 560 - 83.2p_2 \\ &= 1{,}000 + 20p_2 + 560 - 83.2p_2 \\ p_2 &= 24.684 \end{aligned}$$

Thus,

$$p_1 = 25 + 0.5(24.684) = 37.342$$

The corresponding Block 1 output (with price rounded to the nearest penny) is $q_1 = 2{,}000 - 40(37.34) = 506.4$. The Block 2 output is $q_2 = 40p_1 - 40p_2 = 40(37.24) - 40(24.68) = 502.4$.

d. Producer surplus is $PS = 37.34(506.4) + 24.68(502.4) - 11{,}105.68 = \$20{,}202.53$. Consumer surplus is

$$\begin{aligned} CS &= \int_0^{506.4} (50 - 0.025q - 37.34)dq + \int_{506.4}^{1{,}008.8} (50 - 0.025q - 24.68)dq \\ &= \int_0^{506.4} (12.66 - 0.025q)dq + \int_{506.4}^{1{,}008.8} (25.32 - 0.025q)dq \\ &= (12.66q - 0.0125q^2)\big|_0^{506.4} + (25.32q - 0.0125q^2)\big|_{506.4}^{1{,}008.8} \\ &= 6{,}411.024 - 3{,}205.512 + 25{,}542.816 - 12{,}720.968 - 12{,}822.048 + 3{,}205.512 \\ CS &= 6{,}410.824 \end{aligned}$$

Total surplus in the market with block pricing (with two pricing tiers) is \$26,613.35.

Problems

1. A firm with market power faces the demand function $q = 4{,}000 - 40P$. The firm's total cost function is $TC(q) = 10q + 0.001q^2 + 1{,}000$.
   a. If the firm behaves as a single-price monopoly, identify the firm's optimal price and output level.
   b. Demonstrate that the single-price monopolist's profit-maximizing choice of price and output also maximizes producer surplus.
   c. Identify the output level that would maximize total surplus.
   d. Identify the output level that a perfect price-discriminating monopolist would produce.
2. A university's students have demand for credit hours given by $q = 30 - 0.04p$, where p is the price per credit hour.
   a. If the university charges only one price for credit hours irrespective of the number of credit hours a student enrolls for, identify the price per credit hour that would maximize revenue for the university.
   b. If the university adopts a two-tier pricing structure, identify the structure that would maximize revenue for the university.