17. Consider two Bertrand competitors in the market for brie, François and Babette. The cheeses of François and Babette are differentiated, with the demand for François' cheese given by $q_{F}=30-p_{F}+p_{B}$, where $q_{F}$ is the quantity François sells, $p_{F}$ is the price François charges, and $p_{B}$ is the price charged by Babette. The demand for Babette's cheese is similarly given as $q_{B}=30-p_{B}+p_{F}$.
a. Find the Bertrand equilibrium prices and quantities for these two competitors.
b. Now consider a situation in which François sets his price first, and Babette responds. Follow procedures similar to those you used for Stackelberg quantity competition to solve for François' profit-maximizing price, quantity, and profit.
c. Solve for Babette's profit-maximizing price, quantity, and profit.
d. Was François' attempt to seize the first-mover advantage worthwhile?
e. Assuming that each competitor faces a marginal cost and fixed cost of zero, redo part (a) using calculus methods and confirm that your answers are the same as those solved algebraically.
f. Suppose that the demand for François' cheese changes to $q_{F}=30-2 p_{F}+p_{B}$ after he decreases advertising. Solve for the new differentiated Bertrand equilibrium using calculus.
