Solution

4. a. When Wilma chooses Opera, Fred's best response is to also choose Opera. When Wilma chooses Bowling, Fred's best response is to also choose Bowling. Similarly, when Fred chooses Opera, Wilma should also choose Opera, and when Fred chooses Bowling, so should Wilma. Therefore, there are two Nash equilibria, (Opera, Opera) and (Bowling, Bowling).

   b. When Ren chooses Straight, Chuck's best response is to choose Swerve. When Ren chooses Swerve, Chuck should choose Straight. If Chuck chooses Straight, Ren should play Swerve, and when Chuck chooses Swerve, Ren should play Straight. Therefore, there are two Nash equilibria (Swerve, Straight) and (Straight, Swerve).

   c. Assuming that p is the probability that Wilma chooses Opera and q is the probability that Fred chooses Opera, the equilibrium conditions and their solutions are seen to be

$$\begin{aligned} 5q + 0(1-q) &= 0q + 2(1-q) \\ 7q &= 2 \\ q &= \frac{2}{7} \end{aligned}$$

   and

$$\begin{aligned} 2p + 0(1-p) &= 0p + 5(1-p) \\ 7p &= 5 \\ p &= \frac{5}{7} \end{aligned}$$

   There is therefore a mixed-strategy Nash equilibrium in which Wilma chooses Opera with probability $\frac{5}{7}$ and chooses Bowling with probability $\frac{2}{7}$, and Fred chooses Opera with probability $\frac{2}{7}$ and Bowling with probability $\frac{5}{7}$.

   d. Assuming that p is the probability that Ren chooses Straight and q is the probability that Chuck chooses Straight, the equilibrium conditions and their solutions are seen to be

$$\begin{aligned} 0q + 3(1-q) &= q + 2(1-q) \\ 1-q &= q \\ q &= \frac{1}{2} \end{aligned}$$

   and

$$\begin{aligned} 0p + 3(1-p) &= p + 2(1-p) \\ 1-p &= p \\ p &= \frac{1}{2} \end{aligned}$$

   There is therefore a mixed-strategy Nash equilibrium in which each chooses Straight with probability $\frac{1}{2}$ and chooses Swerve with probability $\frac{1}{2}$.