## ∂ 12.2 figure it out

Suppose that two motorcycle manufacturers, Honda and Suzuki, are considering offering 10-year full coverage warranties for their new motorcycles. Although the warranties are expensive to offer, it could be disastrous for one firm if it does not offer a warranty while its competitor does. Let's assume the payoffs for the firms are as follows (profits are in millions of dollars, with Honda's profits in red before the comma and Suzuki's in blue after it):

| Honda \ Suzuki | Offer Warranty | Don't Offer Warranty |
|---|---|---|
| Offer Warranty | 20 , 20 | 120 , 10 |
| Don't Offer Warranty | 10 , 120 | 50 , 50 |

a. If the game is played once, what is the outcome?

b. Suppose the game is repeated three times. Will the outcome change from your answer in (a)? Explain.

c. Now, suppose the game is infinitely repeated and Suzuki and Honda have formed an agreement to not offer warranties to their customers. Each firm plans the use of a grim trigger strategy to encourage compliance with the agreement. At what level of $d$ would Honda be indifferent about keeping the agreement vs. cheating on it? Explain.

d. Suppose that the game's payoffs change as follows:

| Honda \ Suzuki | Offer Warranty | Don't Offer Warranty |
|---|---|---|
| Offer Warranty | 20 , 20 | 120 , 10 |
| Don't Offer Warranty | 30 , 40 | 50 , 50 |

What is the mixed-strategy Nash equilibrium?

### Solution:

a. We can use the check method to solve for the Nash equilibrium in a one-time game:

| Honda \ Suzuki | Offer Warranty | Don't Offer Warranty |
|---|---|---|
| Offer Warranty | ✓ 20 , 20 ✓ | ✓ 120 , 10 |
| Don't Offer Warranty | 10 , 120 ✓ | 50 , 50 |

The Nash equilibrium occurs when both firms offer a warranty. Note that this is not the best cooperative outcome for the game, but it is the only stable equilibrium.

b. If the game is played for three periods, there would be no change in the players' behavior. In the third period, both firms would offer warranties because that is the Nash equilibrium. Knowing this and using backward induction, players will opt to offer warranties in both the second and the first periods as well.

c. Honda's expected payoff from cheating and offering a warranty would be the 120 million from the first period (when cheating) and 20 million for each period after that (because Suzuki will also start offering warranties):

$$\text{Expected payoff from cheating} = 120 + d \times (20) + d^2 \times (20) + d^3 \times (20) + \ldots$$

Honda's expected payoff from following the agreement is earning 50 million each period throughout time:

$$\text{Expected payoff from following agreement} = 50 + d \times (50) + d^2 \times (50) + d^3 \times (50) + \ldots$$

Therefore, Honda will be indifferent between these two options when the payoff streams are equal:

$$120 + d \times (20) + d^2 \times (20) + d^3 \times (20) + \ldots = 50 + d \times (50) + d^2 \times (50) + d^3 \times (50) + \ldots$$

$$d \times (30) + d^2 \times (30) + d^3 \times (30) + \ldots = 70$$

$$d + d^2 + d^3 + \ldots = \frac{7}{3}$$

$$\text{Because } d + d^2 + d^3 + \ldots = \frac{d}{(1-d)} \text{ for any } 0 \leq d < 1\text{:}$$

$$\frac{d}{(1-d)} = \frac{7}{3}$$

$$d = 0.7$$

Therefore, Honda will be indifferent between following the agreement or cheating if $d = 0.7$.

d.

| | | | Suzuki: $q$ Offer Warranty | Suzuki: $1-q$ Don't Offer Warranty |
|---|---|---|---|---|
| Honda | $p$ | Offer Warranty | 20 , 20 | 120 , 10 |
| Honda | $1-p$ | Don't Offer Warranty | 30 , 40 | 50 , 50 |

The equilibrium conditions and their solutions are

$$20q + 120(1-q) = 30q + 50(1-q)$$

$$10q = 70(1-q)$$

$$q = 7 - 7q$$

$$q = \frac{7}{8}$$

## 12.2 figure it out

and

$$20p + 40(1 - p) = 10p + 50(1 - p)$$
$$10p = 10(1 - p)$$
$$p = 1 - p$$
$$p = \frac{1}{2}$$

There is a mixed-strategy Nash equilibrium in which Honda offers a warranty with probability $\frac{1}{2}$ and does not offer a warranty with probability $\frac{1}{2}$, and Suzuki offers a warranty with probability $\frac{7}{8}$ and does not offer a warranty with probability $\frac{1}{8}$.