∂12.3 figure it out

Two firms, GamesRUs (GRU) and PlayThings Incorporated (PTI), are considering a new television advertising campaign for their Christmas gift registries. Because television advertising is expensive, each firm earns greater profit ($50 million) when it does not advertise. If both choose to advertise, very few new customers are gained and each firm earns only $30 million profit. However, if one firm advertises while the other does not, the firm choosing to advertise gets the majority of customers, earning a profit of $70 million, while its competitor only earns $20 million profit.

a. Create a table showing the normal form of this game.

b. List all Nash equilibria.

c. If this game is played sequentially and GRU makes its decision before PTI, what will the outcome be?

d. Is there a first-mover advantage in this case? Explain.

e. Suppose that the game's payoffs change as follows:

		PTI: Advertise	PTI: Don't Advertise
GRU	Advertise	30 , 30	70 , 20
	Don't Advertise	55 , 70	50 , 80

What is the mixed-strategy Nash equilibrium?

Solution:

a. The normal form is shown below (payoffs are in millions of dollars with GRU's profits before the comma and PTI's after):

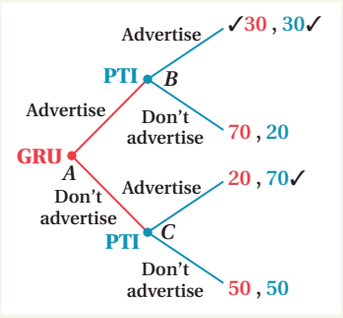
		PTI: Advertise	PTI: Don't Advertise
GRU	Advertise	30 , 30	70 , 20
	Don't Advertise	20 , 70	50 , 50

b. We can use the check method to determine the Nash equilibria. If GRU believes that PTI will advertise, its best move is to also advertise ($30 million > $20 million). If GRU believes that PTI will not advertise, its best move is to advertise ($70 million > $50 million). Therefore, advertising is a dominant strategy for GamesRUs because it is the best strategy no matter what strategy PlayThings Inc. follows. Note, too, that because PTI's payoffs are the same as those of GRU's, advertising is also a dominant strategy for PTI. Therefore, we end up with the following:

GRU \ PTI	Advertise	Don't Advertise
Advertise	✓ 30 , 30 ✓	✓ 70 , 20
Don't Advertise	20 , 70 ✓	50 , 50

Because advertising is a dominant strategy for both firms, the outcome of this game is found in the upper-left portion of the payoff matrix and each firm earns a profit of $30 million. This is the Nash equilibrium because neither firm has an incentive to change its strategy given the strategy of the other.

c. The extensive form of this game (with GamesRUs moving first) is shown in the figure at right:



Using backward induction, we can see that, when GRU chooses to advertise, PTI will also choose to advertise (because $30 million > $20 million). If GRU chooses not to advertise, PTI will still choose to advertise (because $70 million > $50 million). Therefore, if GRU understands how PTI will respond to its strategy, it will also choose to advertise (because $30 million > $20 million).

d. No first-mover advantage exists in this case because both firms have dominant strategies. A dominant strategy is the best strategy to follow *no matter what your opponent does*. Therefore, it is irrelevant if one firm makes its decision before the other firm; each firm will always choose its dominant strategy: to advertise.

e.

GRU \ PTI			q Advertise	$1-q$ Don't Advertise
	p	Advertise	30 , 30	70 , 20
	$1-p$	Don't Advertise	55 , 70	50 , 80

The equilibrium conditions and their solutions are

$$30q + 70(1 - q) = 55q + 50(1 - q)$$
$$25q = 20(1 - q)$$
$$5q = 4 - 4q$$
$$q = \frac{4}{9}$$

and

$$30p + 70(1 - p) = 20p + 80(1 - p)$$
$$10p = 10(1 - p)$$
$$p = 1 - p$$
$$p = \frac{1}{2}$$

There is a mixed-strategy Nash equilibrium in which GRU advertises with probability $\frac{1}{2}$ and does not advertise with probability $\frac{1}{2}$, and PTI advertises with probability $\frac{4}{9}$ and does not advertise with probability $\frac{5}{9}$.