

Solution

13. a. In equilibrium, $Q^S = Q^D$ so that

$$\begin{aligned} Q^D &= 100 - P = -20 + 2P = Q^S \\ 100 - P &= -20 + 2P \\ P_E &= 40 \end{aligned}$$

The equilibrium price for pillows is 40.

b. The equilibrium quantity is

$$Q_E = 100 - P_E = 60$$

c. The equilibrium quantity using the supply equation is

$$Q^S = -20 + 2P_E = 60 = Q_E$$

Hence, $Q^S = Q^D = Q_E$, just like what was obtained in (b).

d. The elasticity of supply or demand can be calculated using the expression

$$E = \frac{1}{\text{slope of the inverse demand curve}} \times \frac{P}{Q}$$

Note that $P = 40$ and $Q = 60$ at the equilibrium point. The slope of the demand curve is -1 , so that

$$E_D = \frac{1}{-1} \times \frac{40}{60} = -\frac{2}{3}$$

whereas the slope of our supply curve is 0.5 , so that

$$E_S = \frac{1}{0.5} \times \frac{40}{60} = \frac{4}{3}$$

The elasticity of demand lies within the interval $-1 < E_D < 0$. Hence, the demand is inelastic. The coefficient of elasticity of supply is elastic. Out of the two, the supply is more elastic.

e. Inverting the demand function, we get

$$\begin{aligned} Q^D &= 100 - P \\ P &= 100 - Q^D \end{aligned}$$

Inverting the supply function yields

$$\begin{aligned} Q^S &= -20 + 2P \\ P &= 0.5Q^S + 10 \end{aligned}$$

At the equilibrium, we get

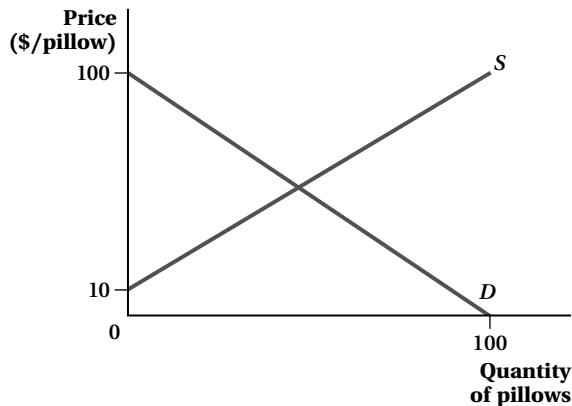
$$\begin{aligned} 100 - Q^D &= \frac{Q^S}{2} + 10 \\ 100 - Q_E &= \frac{1}{2}Q_E + 10 \\ \frac{3}{2}Q_E &= 90 \\ Q_E &= 60 \\ P_E &= 100 - Q_E = 40 \end{aligned}$$

Therefore, the equilibrium point coincides with our previous answer. Since the equilibrium point is the same and since the slopes of both curves are also unchanged, the elasticities will correspond to the previously derived coefficients in part (d).

f. Since $\frac{\partial Q^D}{\partial P} = -1 < 0$, the law of demand holds. Since $\frac{\partial Q^S}{\partial P} = 2 > 0$, the law of supply holds.

g. Using derivatives, the price elasticity of demand is

$$\begin{aligned} E^D &= \frac{\partial Q^D}{\partial P} \frac{P}{Q^D} \\ &= -1 \times \frac{40}{60} \\ &= -\frac{2}{3} \end{aligned}$$



Using derivatives, the price elasticity of supply is

$$\begin{aligned}E^S &= \frac{\partial Q^S}{\partial P} \frac{P}{Q^S} \\ &= 2 \times \frac{40}{60} = \frac{4}{3}\end{aligned}$$

These are the same answers as in part (d), and therefore the price elasticity of supply is more elastic and the price elasticity of demand is more inelastic as determined in that part of the problem.

- h. Since $\frac{\partial Q^S}{\partial P_f} = -0.5 < 0$, we know that the quantity supplied of down pillows decreases as the price of feathers increases, which is the expected relationship given that feathers are inputs to the production of down pillows.
- i. Substituting $P_f = 10$ into the expanded supply curve, we can see that

$$\begin{aligned}Q^S &= -15 + 2P - 0.5(10) \\ &= -20 + 2P\end{aligned}$$

This is the supply curve as given in the problem's setup.

- j. At the new price, $Q^S = -15 + 2P - 0.5(8) = -19 + 2P$. The supply curve has shifted rightward.