Solution

18. a. Consider increasing frog's legs from 1 to 3; that is, by 200%. On the other hand, the price decreases from \$3 to \$1; that is, by 200%. Hence, the price elasticity of demand is

$$E_D = \frac{\%\Delta Q^D}{\%\Delta P} = \frac{200\%}{-200\%} = -1$$

The demand curve is unit-elastic.

b. The revenue function is given by

$$P \times Q = P \times \frac{3}{P} = 3$$

so it does not matter how many frog's legs are sold. As long as the quantity is positive, the revenue is \$3.

c. By inverting $P = \frac{3}{Q^D}$ to standard demand form, we can see that $Q^D = \frac{3}{P}$ or $Q^D = 3P^{-1}$. Now since $\frac{\partial Q^D}{\partial P} = -3P^{-2} = -\frac{3}{P^2} < 0$, we know that the law of demand holds.

d. At a price of \$2, quantity demanded is $\frac{3}{2} = 1.5$. At a price of \$4, quantity demanded is $\frac{3}{4} = 0.75$. From the demand function in standard form, as calculated in part (c), $\frac{\partial Q^D}{\partial P} = -\frac{3}{P^2}$.

At a price of \$2, this equals $-\frac{3}{(2)^2} = -\frac{3}{4}$. At a price of \$4, this equals $-\frac{3}{(4)^2} = -\frac{3}{16}$.

Using derivatives therefore, the price elasticity of demand at a price of \$2 then is

$$\begin{aligned} E^D &= \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} \\ &= -\frac{3}{4} \times \frac{2}{1.5} \\ &= -1 \end{aligned}$$

The price elasticity of demand at a price of \$4 then is

$$\begin{aligned} E^D &= \frac{\partial Q^D}{\partial P}\frac{P}{Q^D} \\ &= -\frac{3}{16} \times \frac{4}{0.75} \\ &= -1 \end{aligned}$$

Note that this is not surprising given that demand was shown in part (a) to be unit-elastic!