# Solution

15. a. The inverse demand function is

$$Q^D = 20 - 2P$$
$$P = 10 - 0.5Q^D$$

The inverse supply curve is

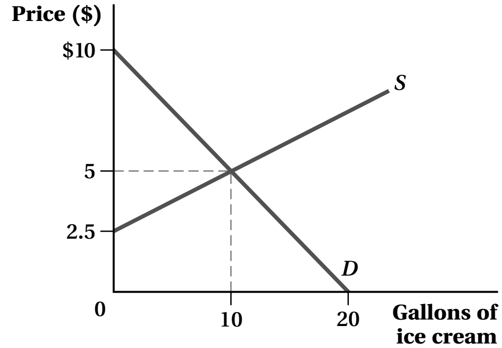
$$Q^S = 4P - 10$$
$$P = 2.5 + 0.25Q^S$$

The equilibrium price is

$$Q^D = 20 - 2P = 4P - 10 = Q^S$$
$$6P = 30$$
$$P^* = \$5$$

The equilibrium quantity is

$$20 - (2 \times 5) = 10 \text{ gallons of ice cream}$$



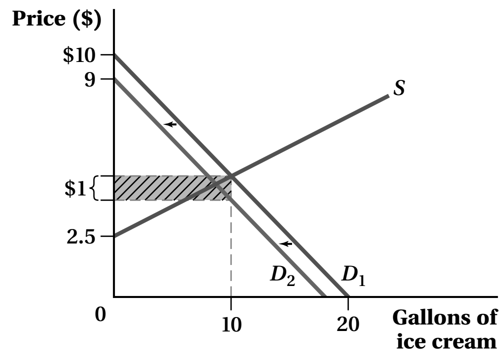

b. The demand curve shifts inward by the amount of the tax. See graph at right.

c. The buyers face a new price $P_S$ + TAX, and sellers sell at $P_S$:

$$Q^D = 20 - 2 \times (P_S + 1) = 4P_S - 10 = Q^S$$
$$6P_S = 28$$
$$P_S \approx 4.67$$

Hence, buyers pay \$5.67 and sellers sell at \$4.67. The quantity sold is

$$4 \times 4.67 - 10 = 8.68$$




d. After the \$1 tax, buyers pay \$5.67 for a gallon of ice cream. After the buyers send in the tax, the sellers only end up with \$4.67 per gallon sold. Therefore, of the \$1 going to the government, approximately 67% of it is coming out of consumers' pockets because their price went up by 67 cents per gallon. The price realized by the suppliers goes down by approximately 33 cents per gallon. The incidence of this tax is 67% on the buyers and 33% on the seller. Hence, buyers bear a proportionately greater burden of the tax. This happens when demand is relatively inelastic when compared to supply, which in this case is relatively elastic.

e. Before the tax, the demand choke price is \$10 per gallon of ice cream. Hence, the consumer surplus is

$$\frac{1}{2} \times (10 - 0) \times (\$10 - \$5) = \$25$$

After the tax, the demand choke price is \$9 per gallon of ice cream. Hence, the consumer surplus is

$$\frac{1}{2} \times (8.68 - 0) \times (\$10 - \$5.67) \approx \$18.79$$

Or if you reduce the choke price and use the gross price of \$4.67, the consumer surplus is still

$$\frac{1}{2} \times (8.68 - 0) \times (\$9 - \$4.67) \approx \$18.79$$

f. Before the tax, the supply choke price is \$2.50 per gallon of ice cream. Hence, the producer surplus is

$$\frac{1}{2} \times (10 - 0) \times (\$5 - \$2.50) \approx \$12.50$$

After the tax, the producer surplus is

$$\frac{1}{2} \times (8.68 - 0) \times (\$4.67 - \$2.50) = 0.5 \times 8.68 \times \$2.17 \approx \$9.42.$$

g. The tax revenue raised by the government is

$$\$1 \times 8.68 \approx \$8.68$$

h. The tax creates the deadweight loss of

$$0.5 \times 1 \times \$1.33 \approx \$0.67$$

i. Recall that deadweight loss is the area between the demand and supply curves over the range of the difference between the old and new quantities, and using the inverse demand and supply equations as derived in part (a)

$$DWL = \int_{8.68}^{10} \big((10 - 0.5Q) - (2.5 + 0.25Q)\big)\ dQ = \int_{8.68}^{10} (7.5 - 0.75Q)\ dQ = \int_{8.68}^{10} 7.5\ dQ - \int_{8.68}^{10} 0.75Q\ dQ$$

$$= [7.5]_{8.68}^{10} - \left[\frac{0.75Q^2}{2}\right]_{8.68}^{10}$$

$$= [7.5(10) - 7.5(8.68)] - \left[\frac{0.75(10)^2}{2} - \frac{0.75(8.68)^2}{2}\right]$$

$$= (75 - 65.1) - (37.5 - 28.25) = 0.65$$

This is within rounding of the \$0.67 as previously calculated.