Solution

7. a. Solving for the equilibrium price and quantity, we get

$$Q^S = 30P - 2{,}000 = 6{,}000 - 20P = Q^D$$
$$50P = 8{,}000$$
$$P^* = \$160$$

Thus, the equilibrium price is \$160. The equilibrium quantity is

$$30P - 2{,}000 = 30 \times 160 - 2{,}000 = 2{,}800$$

Hence, 2,800 divers are being served each year.

b. The demand choke price is

$$Q^D = 6{,}000 - 20P$$
$$0 = 6{,}000 - 20P$$
$$P = \$300$$

Thus, the consumer surplus is

$$\frac{1}{2} \times (2{,}800 - 0) \times (\$300 - \$160) = \$196{,}000$$

c. The supply choke price is

$$Q^S = 30P - 2{,}000$$
$$0 = 30P - 2{,}000$$
$$P \approx \$66.67$$

Thus, the producer surplus is

$$\frac{1}{2} \times (2{,}800 - 0) \times (\$160 - \$66.67) \approx \$130{,}662$$

d. The new demand function is $Q^D = 7{,}000 - 20P$. The new equilibrium price is

$$Q^S = 30P - 2{,}000 = 7{,}000 - 20P = Q^D$$
$$50P = 9{,}000$$
$$P^* = \$180$$

Thus, the new equilibrium price is \$180. The new equilibrium quantity is

$$30 \times 180 - 2{,}000 = 3{,}400$$

Hence, 3,400 divers are being served each year.

The new demand choke price is

$$0 = 7{,}000 - 20P$$
$$P = \$350$$

The new consumer surplus is

$$\frac{1}{2} \times (3{,}400 - 0) \times (\$350 - \$180) = \$289{,}000$$

The new producer surplus is

$$\frac{1}{2} \times (3{,}400 - 0) \times (\$180 - \$66.67) \approx \$192{,}661$$

e. The consumers are better off, as the consumer surplus increases by

$$\$289{,}000 - \$196{,}000 = \$93{,}000$$

f. We can rearrange demand to get inverse demand:

$$Q^D = 6{,}000 - 20P$$
$$20P = 6{,}000 - Q^D$$
$$P = 300 - 0.05Q^D$$

$$CS = \int_0^{2{,}800} (300 - 0.05Q - 160)\ dQ = \int_0^{2{,}800} 140\ dQ - \int_0^{2{,}800} 0.05Q\ dQ = [140Q]_0^{2{,}800} - \left[\frac{0.05Q^2}{2}\right]_0^{2{,}800}$$
$$= [140(2{,}800) - 140(0)] - \left[\frac{0.05(2{,}800)^2}{2} - \frac{0.05(0)^2}{2}\right]$$
$$= (392{,}000 - 0) - (196{,}000 - 0) = 196{,}000$$

This is the same \$196,000 as previously calculated.

g. We can rearrange demand to get inverse supply:

$$Q^S = 30P - 2{,}000$$
$$30P = 2{,}000 + Q^S$$
$$P = \frac{200}{3} + \frac{Q^S}{30}$$

$$PS = \int_0^{2{,}800} \left(160 - \left(\frac{200}{3} + \frac{Q}{30}\right)\right) dQ = \int_0^{2{,}800} \left(\frac{280}{3} - \frac{Q}{30}\right) dQ = \int_0^{2{,}800} \frac{280}{3}\ dQ - \int_0^{2{,}800} \frac{Q}{30}\ dQ$$
$$= \left[\frac{280}{3}Q\right]_0^{2{,}800} - \left[\frac{Q^2}{60}\right]_0^{2{,}800}$$
$$= \left[\frac{280}{3}(2{,}800) - \frac{280}{3}(0)\right] - \left[\frac{(2{,}800)^2}{60} - \frac{(0)^2}{60}\right]$$
$$= (261{,}333.33 - 0) - (130{,}666.67 - 0) = 130{,}666.66$$

Since quantity is measured in hundreds, this is the same \$130,662 as previously calculated within rounding.

h. We can rearrange the new demand to get inverse demand:

$$Q^D = 7{,}000 - 20P$$
$$20P = 7{,}000 - Q^D$$
$$P = 350 - 0.05Q^D$$

$$CS = \int_0^{3{,}400} (350 - 0.05Q - 180)\ dQ = \int_0^{3{,}400} 170\ dQ - \int_0^{3{,}400} 0.05Q\ dQ = [170Q]_0^{3{,}400} - \left[\frac{0.05Q^2}{2}\right]_0^{3{,}400}$$
$$= [170(3{,}400) - 170(0)] - \left[\frac{0.05(3{,}400)^2}{2} - \frac{0.05(0)^2}{2}\right]$$
$$= (578{,}000 - 0) - (289{,}000 - 0) = 289{,}000$$

This is the same \$289,000 as previously calculated.

i. Producer surplus in this case of new demand is

$$PS = \int_0^{3{,}400} \left(180 - \left(\frac{200}{3} + \frac{Q}{30}\right)\right) dQ = \int_0^{3{,}400} \left(\frac{340}{3} - \frac{Q}{30}\right) dQ = \int_0^{3{,}400} \frac{340}{3}\ dQ - \int_0^{3{,}400} \frac{Q}{30}\ dQ$$
$$= \left[\frac{340}{3}Q\right]_0^{3{,}400} - \left[\frac{Q^2}{60}\right]_0^{3{,}400}$$
$$= \left[\frac{340}{3}(3{,}400) - \frac{340}{3}(0)\right] - \left[\frac{(3{,}400)^2}{60} - \frac{(0)^2}{60}\right]$$
$$= (385{,}333.33 - 0) - (192{,}666.67 - 0) = 192{,}666.66$$

This is the same \$192,661 as previously calculated within rounding.

j. The problem asks to calculate consumer surplus under a nonlinear demand curve. As learned in the appendix online, we know that this involves using calculus. With the new demand curve, we need to find the new equilibrium price and quantity. Setting this inverse demand curve equal to inverse supply and solving, we get

$$500 - 0.0001Q^2 = \frac{200}{3} + \frac{Q}{30}$$
$$15{,}000 - 0.003Q^2 = 2{,}000 + Q$$
$$0.003Q^2 + Q - 13{,}000 = 0$$

Using the quadratic formula, we can see that

$$Q = \frac{-1 \pm \sqrt{1^2 - 4(0.003)(-13{,}000)}}{2(0.003)} = (-2{,}254.99,\ 1{,}921.66)$$

Since quantity cannot be negative, we know that the equilibrium quantity is 1,921.66. At this quantity, price (as calculated from the demand side) is $P = 500 - 0.0001(1{,}921.66)^2 = 130.72$. We can now calculate consumer surplus as

$$CS = \int_0^{1{,}921.66} (500 - 0.0001Q^2 - 130.72)\ dQ = \int_0^{1{,}921.66} 369.28\ dQ - \int_0^{1{,}921.66} 0.0001Q^2\ dQ$$
$$= [369.28Q]_0^{1{,}921.66} - \left[\frac{0.0001Q^3}{3}\right]_0^{1{,}921.66}$$
$$= [369.28(1{,}921.66) - 369.28(0)] - \left[\frac{0.0001(1{,}921.66)^3}{3} - \frac{0.0001(0)^3}{3}\right]$$
$$= (709{,}630.6 - 0) - (236{,}542.1 - 0) = 473088.5$$

Consumer surplus under this nonlinear demand curve is then \$473,088.50.