∂ 7. A consumer's utility function is given by $U = XY$, where $MU_X = Y$ and $MU_Y = X$.

a. What is the utility derived from 1 unit of $X$ and 2 units of $Y$? What is the utility derived from 2 units of $X$ and 1 unit of $Y$? What is the utility derived from 5 units of $X$ and 2 units of $Y$?

b. How does the consumer rank the following bundles?

| Bundle | Quantity of X | Quantity of Y |
|---|---|---|
| A | 2 | 2 |
| B | 10 | 0 |
| C | 1 | 5 |
| D | 3 | 2 |
| E | 2 | 3 |

c. Graph an indifference curve that shows the bundles of $X$ and $Y$ for which $U = 6$ and $U = 8$. Is the "more is better" assumption satisfied for $X$ and $Y$?

d. What are $MU_X$ and $MU_Y$ for the following bundles?

| Bundle | Quantity of X | Quantity of Y |
|---|---|---|
| F | 1 | 2 |
| G | 2 | 2 |
| H | 1 | 3 |

e. Does $MU_X$ diminish, stay constant, or increase as $X$ increases? (*Hint*: You must keep the values of all other variables fixed.)

f. Given the utility function in this problem, show that the marginal utilities are as given using calculus.

g. Relate the shape of the consumer's indifference curves to his or her marginal rate of substitution.

h. Suppose that the price of good $X$ is \$2, the price of good $Y$ is \$4, and the consumer's income is \$80. Use a Lagrangian to solve the constrained utility-maximization problem for the consumer.