Solution

17. a. Anthony's marginal rate of substitution can be determined from his marginal utility for each good:

$$MRS_{LG} = \frac{MU_L}{MU_G} = \frac{0.5L^{-0.5}G^{0.5}}{0.5L^{0.5}G^{-0.5}}$$
$$= \frac{G}{L} = \frac{2}{1}$$
$$G^* = 2L^*$$

Anthony's optimal bundle must also lie on his budget constraint, $2L + G = 30$. Substituting the relation from the tangency condition gives

$$2L^* + 2L^* = 30$$
$$4L^* = 30$$
$$L^* = 7.5$$

and

$$G^* = 15$$

The optimal consumption bundle is $(L^*, G^*) = (7.5, 15)$, and this will give Anthony utility $U = (7.5)^{0.5}(15)^{0.5} = 10.6$.

b. If there is a doubling in the price of guitar picks to $P_G = \$2$, then from the tangency condition:

$$MRS_{LG} = \frac{P_L}{P_G}$$
$$\frac{G}{L} = \frac{2}{2}$$
$$G^* = L^*$$

Anthony will want to consume the two goods in equal quantities. In order to maintain utility at $U = 10.6$

$$U(L,G) = L^{0.5}G^{0.5}$$
$$10.6 = L^{0.5}G^{0.5}$$

with $G^* = L^* = k$

$$10.6 = k^{0.5}k^{0.5}$$
$$10.6 = k$$

Anthony will consume the bundle $(L^*, G^*) = (10.6, 10.6)$, which at the new price for guitar picks will cost

$$(2)(10.6) + (2)(10.6) = \$42.40$$

Anthony will require income $I' = \$42.40$ in order to maintain the same level of utility.

c. Marginal utilities can be calculated as partial derivatives of the utility function. Since $U(L,G) = L^{0.5}G^{0.5}$, the marginal utility of fishing lures is $MU_L = \frac{\partial U}{\partial L} = 0.5L^{-0.5}G^{0.5}$, and the marginal utility of guitar picks is $MU_G = \frac{\partial U}{\partial G} = 0.5L^{0.5}G^{-0.5}$.

d. The Lagrangian is

$$\mathcal{L}(L,G,\lambda) = L^{0.5}G^{0.5} + \lambda[30 - 2L - G]$$

The first-order conditions (partial derivatives of this equation with respect to L, G, and the Lagrange multiplier λ, respectively) are

$$\frac{\partial \mathcal{L}}{\partial L} = 0.5L^{-0.5}G^{0.5} - \lambda(2) = 0$$
$$\frac{\partial \mathcal{L}}{\partial G} = 0.5L^{0.5}G^{-0.5} - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 30 - 2L - G = 0$$

Solving the first two equations for λ and setting them equal to each other, we find

$$\lambda = \frac{0.5L^{-0.5}G^{0.5}}{2} = \frac{0.5L^{0.5}G^{-0.5}}{1}$$

Rearranging, we see that

$$0.5L^{-0.5}G^{0.5} = L^{0.5}G^{-0.5}$$
$$L = 0.5G$$

We can combine this with the third of the first-order conditions that tells us the budget constraint relationship:

$$30 = 2(0.5G) + G$$
$$2G = 30$$
$$G^* = 15$$
$$L^* = 0.5(15) = 7.5$$

Note that this is the same answer as that to part (a)!

e. (i) Anthony's constrained optimization problem is

$$\min_{L,G} 2L + 2G \text{ s.t. } 10.6 = L^{0.5}G^{0.5}$$

Note that 10.6 is the utility level corresponding to the optimal consumption bundle as solved for in part (a).

(ii) The Lagrangian is

$$\mathcal{L}(L,G,\lambda) = 2L + 2G + \lambda[10.6 - L^{0.5}G^{0.5}]$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial L} = 2 - \lambda 0.5L^{-0.5}G^{0.5} = 0$$
$$\frac{\partial \mathcal{L}}{\partial G} = 2 - \lambda 0.5L^{0.5}G^{-0.5} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 10.6 - L^{0.5}G^{0.5} = 0$$

Solve for λ in the first two conditions and set these two expressions equal to one another:

$$\lambda = \frac{2}{0.5L^{-0.5}G^{0.5}} = \frac{2}{0.5L^{0.5}G^{-0.5}}$$
$$L = G$$

Substituting into the third constraint yields

$$10.6 - G^{0.5}G^{0.5} = 0$$
$$G^* = 10.6$$

Since $L = G$ from above, we get

$$L^* = 10.6$$

Thus, the minimum expenditure is \$2(10.6) + \$2(10.6) = \$42.4. Note that this is the same answer as the answer to part (b)!

f. The Lagrangian is now

$$\mathcal{L}(L,G,\lambda) = L^{0.5}G^{0.5} + \lambda[30 - 2L - 2G]$$

The first-order conditions (partial derivatives of this equation with respect to L, G, and the Lagrange multiplier λ, respectively) are

$$\frac{\partial \mathcal{L}}{\partial L} = 0.5L^{-0.5}G^{0.5} - \lambda(2) = 0$$
$$\frac{\partial \mathcal{L}}{\partial G} = 0.5L^{0.5}G^{-0.5} - \lambda(2) = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 30 - 2L - 2G = 0$$

Solving the first two equations for λ and setting them equal to each other, we find

$$\lambda = \frac{0.5L^{-0.5}G^{0.5}}{2} = \frac{0.5L^{0.5}G^{-0.5}}{2}$$

Rearranging, we see that

$$0.5L^{-0.5}G^{0.5} = 0.5L^{0.5}G^{-0.5}$$

$$L = G$$

We can combine this with the third of the first-order conditions that tells us the budget constraint relationship:

$$30 = 2(G) + 2G$$

$$4G = 30$$

$$G^{*} = 7.5$$

Since $L = G$ from above, we get

$$L^{*} = 7.5$$