Solution

7\. a.

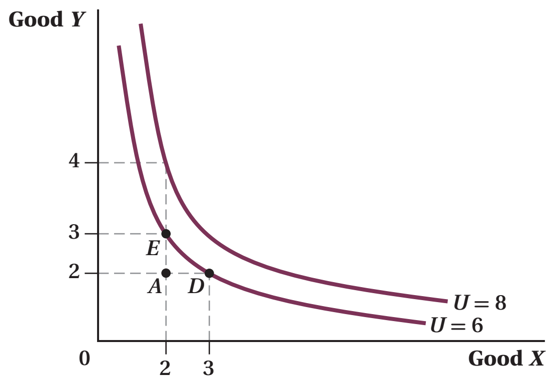
$$U(1, 2) = (1)(2) = 2$$
$$U(2, 1) = (2)(1) = 2$$
$$U(5, 2) = (5)(2) = 10$$

b.

Bundle	Quantity of *X*	Quantity of *Y*	Utility
A	2	2	4
B	10	0	0
C	1	5	5
D	3	2	6
E	2	3	6

From the table, $U(D) = U(E) > U(C) > U(A) > U(B)$ and we have the ranking ($>$ denotes strictly preferred, $\sim$ denotes indifferent)

$$D \sim E > C > A > B$$

c.


The "more is better" assumption is satisfied.

d.

Bundle	Quantity of *X*	Quantity of *Y*	MU_x	MU_Y
F	1	2	2	1
G	2	2	2	2
H	1	3	3	1

e. Comparing bundle F and G, $MU_X = 2$ for both bundles. MU_X is constant.

f. Marginal utilities can be calculated as partial derivatives of the utility function. Since $U = XY$, the marginal utility of X is $MU_X = \frac{\partial U}{\partial X} = Y$, and the marginal utility of Y is $MU_Y = \frac{\partial U}{\partial Y} = X$.

g. Indifference curves corresponding to the consumer's utility function are convex to the origin. The slopes of these indifference curves therefore change as the consumer gives up Y to get more X. Specifically, $MRS_{XY} = \frac{MU_X}{MU_Y} = \frac{Y}{X}$. This MRS decreases as the consumer gives up Y to obtain more X.

h. The Lagrangian is

$$\mathcal{L}(X,Y,\lambda) = XY + \lambda[80 - 2X - 4Y]$$

The first-order conditions (partial derivatives of this equation with respect to X, Y, and the Lagrange multiplier λ, respectively) are

$$\frac{\partial \mathcal{L}}{\partial X} = Y - \lambda(2) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = X - \lambda(4) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 80 - 2X - 4Y = 0$$

Solving the first two equations for λ and setting them equal to each other, we find

$$\lambda = \frac{Y}{2} = \frac{X}{4}$$

Rearranging, we see that

$$4Y = 2X$$

$$X = 2Y$$

We can combine this with the third of the first-order conditions that tells us the budget constraint relationship:

$$80 = 2(2Y) + 4Y$$

$$8Y = 80$$

$$Y = 10$$

$$X = 2(10) = 20$$