∂ 4.1 figure it out

Mariah consumes music downloads (M) and concert tickets (C). Her utility function is given by $U = 0.5M^2 + 2C^2$, where $MU_M = M$ and $MU_C = 4C$.

a. Write an equation for MRS_{MC}

b. Would bundles of ($M = 4$ and $C = 1$) and ($M = 2$ and $C = 2$) be on the same indifference curve? How do you know?

c. Calculate MRS_{MC} when $M = 4$ and $C = 1$ and when $M = 2$ and $C = 2$.

d. Based on your answers to question b, are Mariah's indifference curves convex? (*Hint*: Does MRS_{MC} fall as M rises?)

e. Given Mariah's utility function, show that the marginal utilities of music downloads and concert tickets are as given using calculus.

f. Use calculus to show whether or not Mariah's indifference curves are convex and confirm that your answer is the same as in part (d).

g. Determine whether Mariah's preferences can also be represented by $U = 2M^2 + 8C^2$.

Solution:

a. We know that the marginal rate of substitution MRS_{MC} equals MU_M/MU_C

We are told that $MU_M = M$ and that $MU_C = 4C$. Thus, $MRS_{MC} = \frac{MU_M}{MU_C} = \frac{M}{4C}$.

b. For bundles to lie on the same indifference curve, they must provide the same level of utility to the consumer. Therefore we need to calculate Mariah's level of utility for the bundles of ($M = 4$ and $C = 1$) and ($M = 2$ and $C = 2$):

When $M = 4$ and $C = 1$, $U = 0.5(4)^2 + 2(1)^2 = 0.5(16) + 2(1) = 8 + 2 = 10$

When $M = 2$ and $C = 2$, $U = 0.5(2)^2 + 2(2)^2 = 0.5(4) + 2(4) = 2 + 8 = 10$

Each bundle provides Mariah with the same level of utility, so they must lie on the same indifference curve.

c. and d. To determine if Mariah's indifference curve is convex, we need to calculate MRS_{MC} at both bundles. Then we can see if MRS_{MC} falls as we move down along the indifference curve (i.e., as M increases and C decreases).

$$\text{When } M = 2 \text{ and } C = 2,\ MRS_{MC} = \frac{2}{(4)(2)} = \frac{2}{8} = \frac{1}{4} = 0.25$$

$$\text{When } M = 4 \text{ and } C = 1,\ MRS_{MC} = \frac{4}{(4)(1)} = \frac{4}{4} = 1$$

These calculations reveal that, holding utility constant, when music downloads rise from 2 to 4, the MRS_{MC} rises from 0.25 to 1. This means that as Mariah consumes more music downloads and fewer concert tickets, she actually becomes *more* willing to trade concert tickets for additional music downloads! Most consumers would not behave in this way. This means that the indifference curve becomes steeper as M rises, not flatter. In other words, this indifference curve will be concave to the origin rather than convex, violating the fourth characteristic of indifference curves listed above.

e. Marginal utilities can be calculated as partial derivatives of the utility function. Since $U = 0.5M^2 + 2C^2$, the marginal utility of M is $MU_M = \dfrac{\partial U}{\partial M} = 0.5(2)M^{2-1} = M$, and the marginal utility of C is $MU_C = \dfrac{\partial U}{\partial C} = 2(2)C^{2-1} = 4C$.

f. A way to check concavity versus convexity is to take the second derivative of the utility function and compare it to zero. The second derivative of the utility function with respect to M is $\dfrac{\partial^2 U}{\partial M^2} = \dfrac{\partial MU_M}{\partial M} = 1$, which is more than zero. Similarly, the second derivative of the utility function with respect to C is $\dfrac{\partial^2 U}{\partial C^2} = \dfrac{\partial MU_C}{\partial C} = 4$, which is more than zero. This indicates that Mariah's indifference curve is concave to the origin, as you showed in part (d). (Note that we could also show concavity by taking the partial derivative of MRS_{MC} with respect to M to illustrate how the marginal rate of substitution changes as M rises. Here, $\dfrac{\partial MRS_{MC}}{\partial M} = \dfrac{1}{4C}$. As C decreases, this increases, which is consistent with concavity to the origin.)

g. From the online appendix, we know that monotonic transformations of utility functions are representations of the same preferences and that in these cases, the marginal rate of substitution will be equal. You found that the marginal rate of substitution for the original case is $MRS_{MC} = \dfrac{MU_M}{MU_C} = \dfrac{M}{4C}$. Now, we want to check the marginal rate of substitution for the new utility function in this part of the problem. Here, $MU_M = \dfrac{\partial U}{\partial M} = 2(2)M^{2-1} = 4M$ and $MU_C = \dfrac{\partial U}{\partial C} = 8(2)C^{2-1} = 16C$. The marginal rate of substitution then is $MRS_{MC} = \dfrac{MU_M}{MU_C} = \dfrac{4M}{16C} = \dfrac{M}{4C}$. Since this is equal to the marginal rate of substitution for the original problem, we know that, yes, Mariah's preferences can be represented alternately by the given form.