∂ 4.4 figure it out

Suppose Antonio gets utility from consuming two goods, burgers and fries. His utility function is given by

$$U = \sqrt{BF} = B^{0.5}F^{0.5}$$

where B is the amount of burgers he eats and F the servings of fries. Antonio's marginal utility of a burger $MU_B = 0.5B^{-0.5}F^{0.5}$, and his marginal utility of an order of fries $MU_F = 0.5B^{0.5}F^{-0.5}$. Antonio's income is \$20, and the prices of burgers and fries are \$5 and \$2, respectively.

a. What are Antonio's utility-maximizing quantities of burgers and fries?

b. Suppose that Antonio's utility function is instead $U = 0.5B + 0.5F$

(i) Set up a Lagrangian and derive the first-order conditions for the maximization problem.

(ii) Is there a solution to the first-order conditions?

c. What is the solution to the maximization problem?

Solution:

a. We know that the optimal solution to the consumer's maximization problem sets the marginal rate of substitution—the ratio of the goods' marginal utilities—equal to the goods' price ratio:

$$MRS_{BF} = \frac{MU_B}{MU_F} = \frac{P_B}{P_F}$$

where MU_B and MU_F are the marginal utilities of burgers and fries, respectively. P_B and P_F are the goods' prices. Therefore, to find the utility-maximizing quantities of burgers and fries, we set the ratio of marginal utilities equal to the goods' price ratio and simplify:

$$\frac{MU_B}{MU_F} = \frac{P_B}{P_F}$$

$$\frac{0.5B^{-0.5}F^{0.5}}{0.5B^{0.5}F^{-0.5}} = \frac{5}{2}$$

$$\frac{0.5F^{0.5}F^{0.5}}{0.5B^{0.5}B^{0.5}} = \frac{5}{2}$$

$$\frac{F}{B} = \frac{5}{2}$$

$$2F = 5B$$

$$F = 2.5B$$

This condition tells us that Antonio maximizes his utility when he consumes fries to burgers at a 5 to 2 ratio. We now know the ratio of the optimal quantities, but do not yet know exactly what quantities Antonio will choose to consume. To figure that out, we can use the budget constraint, which pins down the total amount Antonio can spend, and therefore the total quantities of each good he can consume.

Antonio's budget constraint can be written as

$$\text{Income} = P_F F + P_B B, \text{ or}$$

$$B = \frac{\text{Income}}{P_B} - \frac{P_F}{P_B}F$$

Substituting in the values from the problem gives

$$B = \frac{20}{5} - \frac{2}{5}F$$

$$B = 4 - 0.4F$$

Now, we can substitute the utility-maximization condition $F = 2.5B$ into the budget constraint to find the quantity of burgers Antonio will consume:

$$B = 4 - 0.4F$$

$$B = 4 - 0.4(2.5B)$$

$$B = 4 - B$$

$$B = 2$$

And because $F = 2.5B$, then $F = 5$.

Therefore, given his budget constraint, Antonio maximizes his utility by consuming 2 burgers and 5 servings of fries.

b. (i) We can write Antonio's problem as a Lagrangian and take the first-order conditions:

$$\max_{B,F,\lambda} \mathcal{L}(B,F,\lambda) = 0.5B + 0.5F + \lambda(20 - 5B - 2F)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial B} = 0.5 - 5\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial F} = 0.5 - 2\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 20 - 5B - 2F = 0$$

(ii) Solve for λ in the first two FOCs:

$$\lambda = 0.1 \text{ and } \lambda = 0.25$$

Clearly, there is a mathematical contradiction here and therefore a solution to these first-order conditions does not exist.

c. The solution here is a corner solution and the method therefore is as discussed in the online appendix. Marginal utility of burgers can be derived using calculus: $MU_B = \frac{\partial U}{\partial B} = 0.5$. Similarly, the marginal utility of fries is $MU_F = \frac{\partial U}{\partial F} = 0.5$. The marginal rate of substitution of burgers to fries then is $MRS_{BF} = \frac{MU_B}{MU_F} = \frac{0.5}{0.5} = 1$. The slope of the budget constraint (in absolute value form) is the price ratio: $\frac{P_B}{P_F} = \frac{5}{2} = 2.5$. Setting this equal to the marginal rate of substitution would imply a mathematical contradiction as with the Lagrangian approach.

Graphically, with burgers on the x-axis and fries on the y-axis (as in the figure on the right), we see that the budget constraint is much steeper than the indifference curves that correspond to the given utility function (which incidentally are straight lines since this is the special case of perfect substitutes). Because the goal is utility maximization, we can shift a representative indifference curve in the northeast direction as far as possible given the budget constraint. It then becomes obvious that Antonio should consume all fries and no burgers to maximize his utility. It also becomes obvious that this is a corner solution where MRS does not equal the price ratio. So, how many fries specifically should he eat? Turning to the budget constraint with zero substituted for burgers, we see that

$$20 = 5(0) + 2F$$
$$20 = 2F$$
$$F = 10$$

The best that Antonio can do is to eat 10 fries and 0 burgers. Since these are "servings" of fries, Antonio must be hungry!