∂ 5.3 figure it out

Pavlo eats cakes and pies. His income is \$20, and when cakes and pies both cost \$1, Pavlo consumes 4 cakes and 16 pies (point A in Figure A). But when the price of pies rises to \$2, Pavlo consumes 12 cakes and 4 pies (point B).

a. Why does the budget constraint rotate as it does in response to the increase in the price of pies?

b. Trace the diagram on a piece of paper. On your diagram, separate the change in the consumption of pies into the substitution effect and the income effect. Which is larger?

c. Are pies a normal or inferior good? How do you know? Are cakes a normal or inferior good? How do you know?

d. Suppose that when cakes and pies both cost \$1, Pavlo's utility function is $U(C,P) = C^{0.2}P^{0.8}$, where C is cakes and P is pies. Use calculus to show Pavlo's optimal consumption bundle of cakes and pies and utility at these original prices.

e. Suppose that when cakes are \$1 and pies are \$2, Pavlo's utility function is $U(C,P) = C^{0.75}P^{0.5}$. Use calculus to show Pavlo's optimal consumption bundle of cakes and pies and utility at these new prices.

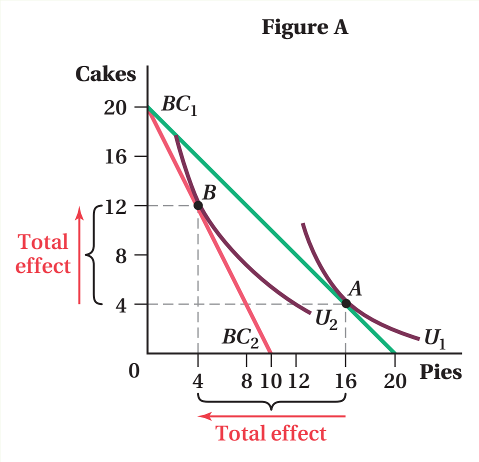
f. Use calculus to decompose the effect of the price change on Pavlo's consumption into total, substitution, and income effects for cakes and pies.

g. Under these utility functions, determine whether pies are a normal or inferior good using calculus. Determine whether cakes are a normal or inferior good using calculus.

h. Are your answers to parts (f) and (g) the same as those for parts (b) and (c)? Why or why not?

i. Assuming that Pavlo's utility function is $U(C,P) = C^{0.75}P^{0.5}$, derive Pavlo's Marshallian demand curve for pies, and show that the Law of Demand is satisfied using calculus.

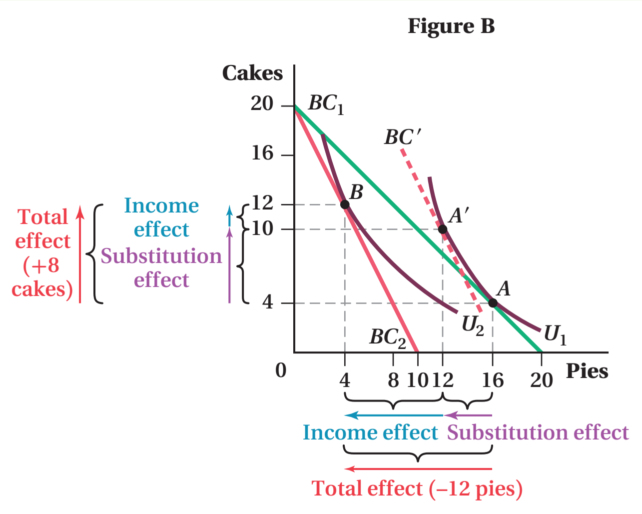
j. Assuming that Pavlo's utility function is $U(C,P) = C^{0.75}P^{0.5}$, derive Pavlo's Hicksian demand curve for pies using calculus.



Solution:

a. The price of cakes hasn't changed, so Pavlo can still buy 20 cakes if he spends his \$20 all on cakes (the y-intercept). However, at \$2 per pie, Pavlo can now afford to buy only 10 pies instead of 20.

b. The substitution effect is measured by changing the ratio of the prices of the goods but holding utility constant (Figure B). Therefore, it must be measured along one indifference curve. To determine the substitution effect of a price change in pies, you need to shift the post-price-change budget constraint BC_2 out until it is tangent to Pavlo's initial indifference



curve U_1. The easiest way to do this is to draw a new budget line BC' that is parallel to the new budget constraint (thus changing the ratio of the cake and pie prices) but tangent to U_1 (thus holding utility constant). Label the point of tangency A'. Point A' is the bundle Pavlo would buy if the relative prices of cakes and pies changed as they did, but he experienced no change in purchasing power. When the price of pies rises, Pavlo would substitute away from buying pies and buy more cakes.

The income effect is the part of the total change in quantities consumed that is due to the change in Pavlo's buying power after the price of pies changes. This is reflected in the shift from point A' on budget constraint BC' to point B on budget constraint BC_2. (These budget constraints are parallel because the income effect is measured holding relative prices constant.)

For pies, the income effect is larger than the substitution effect. The substitution effect leads Pavlo to purchase 4 fewer pies (from 16 to 12), while the income effect further reduces his consumption by 8 pies (from 12 to 4).

c. Pies are a normal good because Pavlo purchases fewer pies (4 instead of 12) when the purchasing power of his income falls due to the price increase. However, cakes are an inferior good because the fall in purchasing power actually leads to a rise in cake consumption.

d. We need to solve Pavlo's original constrained optimization problem $\max_{C,P} C^{0.2}P^{0.8}$ s.t. $20 = C + P$ using the Lagrangian approach. The Lagrangian corresponding to this is

$$\max_{C,P,\lambda} \mathcal{L}(C,P,\lambda) = C^{0.2}P^{0.8} + \lambda(20 - C - P)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = 0.2C^{-0.8}P^{0.8} - \lambda(1) = 0$$
$$\frac{\partial \mathcal{L}}{\partial P} = 0.8C^{0.2}P^{-0.2} - \lambda(1) = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 20 - C - P = 0$$

This is a system of three equations with three unknowns (C, P, and λ). The solution (C,P) is the allocation that we are interested in. Combining the first two equations, we see that

$$\lambda = 0.2C^{-0.8}P^{0.8} = 0.8C^{0.2}P^{-0.2}$$

Therefore, $P = 4C$.

Combining this with the third of the first-order conditions, we get

$$20 = C + P$$
$$20 = C + 4C$$
$$C = 4$$
$$P = 4(4) = 16$$

As in the original setup of the problem, Pavlo should buy 4 cakes and 16 pies at these prices. We also want to find Pavlo's level of utility at this original allocation: $U(4,16) = 4^{0.2}16^{0.8} \approx 12.13$.

e. We need to solve Pavlo's new constrained optimization problem $\max_{C,P} C^{0.75}P^{0.5}$ s.t. $20 = C + 2P$ using the Lagrangian approach. The Lagrangian corresponding to this is

$$\max_{C,P,\lambda} \mathcal{L}(C,P,\lambda) = C^{0.75}P^{0.5} + \lambda(20 - C - 2P)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = 0.75C^{-0.25}P^{0.5} - \lambda(1) = 0$$
$$\frac{\partial \mathcal{L}}{\partial P} = 0.5C^{0.75}P^{-0.5} - \lambda(2) = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 20 - C - 2P = 0$$

This is a system of three equations with three unknowns (C, P, and λ). The solution (C,P) is the allocation that we are interested in. Combining the first two equations, we see that

$$\lambda = \frac{0.75C^{-0.25}P^{0.5}}{1} = \frac{0.5C^{0.75}P^{-0.5}}{2}$$

Therefore, $C = 3P$.

Combining this with the third of the first-order conditions, we get

$$20 = C + 2P$$
$$20 = 3P + 2P$$
$$P = 4$$
$$C = 3(4) = 12$$

As in the original setup of the problem, Pavlo should buy 12 cakes and 4 pies after the price change.

f. To decompose this total effect into income and substitution effects, we can solve the expenditure minimization problem $\min_{C,P} C + 2P$ s.t. $12.13 = C^{0.2}P^{0.8}$. The Lagrangian corresponding to this is

$$\mathcal{L}(C,P,\lambda) = C + 2P + \lambda(12.13 - C^{0.2}P^{0.8})$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = 1 - \lambda 0.2C^{-0.8}P^{0.8} = 0$$
$$\frac{\partial \mathcal{L}}{\partial P} = 2 - \lambda 0.8C^{0.2}P^{-0.2} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 12.13 - C^{0.2}P^{0.8} = 0$$

This is now a system of three equations with three unknowns that can be solved for C, P, and λ. C and P give the allocation that will allow us to separate the substitution and income effects:

$$\lambda = \frac{1}{0.2C^{-0.8}P^{0.8}} = \frac{2}{0.8C^{0.2}P^{-0.2}}$$

Therefore, $P = 2C$.

Combining this with the third of the first-order conditions, we get

$$\begin{aligned} 12.13 &= C^{0.2}P^{0.8} \\ 12.13 &= C^{0.2}(2C)^{0.8} \\ 12.13 &= 2^{0.8}C^{0.2}C^{0.8} \\ 12.13 &= 2^{0.8}C \\ C &= 6.97 \\ P &= 2(6.97) = 13.94 \end{aligned}$$

The substitution effect for pies therefore is the difference between 16 and 13.94 (a decrease of 2.06 pies), and the income effect for pies is the difference between 13.94 and 4 (a decrease of 9.94 pies). The total effect is thus a decrease of 12 pies. For cakes, we find that the substitution effect is a positive 2.97 cakes (the difference between the original 4 and the 6.97 that we find here). The income effect is also positive (here, it is 5.03, the difference between 6.97 and 12). The total effect is an increase of 8 cakes.

g. Pies are a normal good for Pavlo since the income and substitution effects are both moving in the negative direction for the price increase. Cakes, on the other hand, are an inferior good for Pavlo.

h. The answers using calculus are qualitatively, but not numerically, the same as those in parts (b) and (c). The reason is that the particular curvatures of the indifference curves are different between the functional forms and those drawn in Figures A and B. This illustrates just how important preferences are!

i. Pavlo's constrained optimization problem is $\max_{C,P} C^{0.75}P^{0.5}$ s.t. $\overline{I} = p_P P + \overline{p_C}C$. The Lagrangian corresponding to this is

$$\max_{C,P,\lambda} \mathcal{L}(C,P,\lambda) = C^{0.75}P^{0.5} + \lambda(\overline{I} - p_P P - \overline{p_C}C)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = 0.75C^{-0.25}P^{0.5} - \lambda(\overline{p_C}) = 0$$
$$\frac{\partial \mathcal{L}}{\partial P} = 0.5C^{0.75}P^{-0.5} - \lambda(p_P) = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{I} - p_P P - \overline{p_C}C = 0$$

Combining the first two equations, we see that

$$\lambda = \frac{0.75C^{-0.25}P^{0.5}}{\overline{p_C}} = \frac{0.5C^{0.75}P^{-0.5}}{p_P}$$

Therefore, $2\overline{p_C}C = 3p_P P$ or $C = \frac{3p_P P}{2\overline{p_C}}$.

Combining this with the third of the first-order conditions, we get

$$\overline{I} = p_P P + \overline{p_C}C$$
$$\overline{I} = p_P P + \overline{p_C}\left(\frac{3p_P P}{2\overline{p_C}}\right)$$

Pavlo's Marshallian demand curve for pies is $P = \frac{\overline{I}}{2.5p_P}$. Since $\frac{\partial P}{\partial p_P} = -\frac{\overline{I}}{2.5p_P^2} < 0$, the Law of Demand stating that demand is decreasing in price holds.

j. Pavlo's constrained optimization problem is $\min_{C,P} p_P P + \overline{p_C}C$ s.t. $\overline{U} = C^{0.75}P^{0.5}$. The Lagrangian corresponding to this is

$$\min_{C,P,\lambda} \mathcal{L}(C,P,\lambda) = p_P P + \overline{p_C}C + \lambda(\overline{U} - C^{0.75}P^{0.5})$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C} = \overline{p_C} - \lambda(0.75C^{-0.25}P^{0.5}) = 0$$
$$\frac{\partial \mathcal{L}}{\partial P} = p_P - \lambda(0.5C^{0.75}P^{-0.5}) = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{U} - C^{0.75}P^{0.5} = 0$$

Combining the first two equations, we see that

$$\lambda = \frac{\overline{p_C}}{0.75C^{-0.25}P^{0.5}} = \frac{p_P}{0.5C^{0.75}P^{-0.5}}$$

Therefore, $2\overline{p_C}C = 3p_PP$ or $C = \frac{3p_PP}{2\overline{p_C}}$

Combining this with the third of the first-order conditions, we get

$$\overline{U} = C^{0.75}P^{0.5}$$
$$\overline{U} = \left(\frac{3p_PP}{2\overline{p_C}}\right)^{0.75}P^{0.5} = P^{1.25}\left(\frac{3p_P}{2\overline{p_C}}\right)^{0.75}$$

Pavlo's Hicksian demand curve for pies then is

$$P^{1.25} = \overline{U}\left(\frac{2\overline{p_C}}{3p_P}\right)^{0.75}$$
$$P = \overline{U}^{0.8}\left(\frac{2\overline{p_C}}{3p_P}\right)^{0.6}$$