Solution

13. a. Carmen's indifference curves are straight, parallel lines. For her, watching a movie and seeing a basketball game are perfect substitutes. Carmen's utility function can be described as $U = X + 2Y$, where X denotes the basketball games and Y denotes the movies.

b. The optimal consumption bundle is to buy 5 basketball games in order to reach the highest feasible utility curve U_4, given the budget constraint.

c. We need to solve Carmen's original constrained optimization problem $\max_{X,Y} 5XY$ s.t. $90 = 18X + 10Y$ using the Lagrangian approach. The Lagrangian corresponding to this is

$$\max_{X,Y,\lambda} \mathcal{L}(X,Y,\lambda) = 5XY + \lambda(90 - 18X - 10Y)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = 5Y - \lambda(18) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 5X - \lambda(10) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 90 - 18X - 10Y = 0$$

This is a system of three equations with three unknowns (X, Y, and λ). The solution (X,Y) is the allocation that we are interested in. Combining the first two equations, we see that

$$\lambda = \frac{5Y}{18} = \frac{5X}{10}$$

Therefore, $18X = 10Y$.

Combining this with the third of the first-order conditions, we get

$$90 = 18X + (18X)$$

$$90 = 36X$$

$$X = 2.5$$

Since $Y = \frac{9X}{5}$:

$$Y = \frac{9(2.5)}{5} = 4.5$$

Carmen should buy 2.5 units of X and 4.5 units of Y at these prices.

We also want to find Carmen's level of utility at this original allocation: $U(2.5,4.5) = 5(2.5)(4.5) = 56.25$.

d. We need to solve Carmen's new constrained optimization problem using the Lagrangian approach: $\max_{X,Y} 5XY$ s.t. $90 = 20X + 10Y$. The Lagrangian corresponding to this is

$$\max_{X,Y,\lambda} \mathcal{L}(X,Y,\lambda) = 5XY + \lambda(90 - 20X - 10Y)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = 5Y - \lambda(20) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 5X - \lambda(10) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 90 - 20X - 10Y = 0$$

This is a system of three equations with three unknowns (X, Y, and λ). The solution (X,Y) is the allocation that we are interested in. Combining the first two equations, we see that

$$\lambda = \frac{5Y}{20} = \frac{5X}{10}$$

Therefore, $2X = Y$.

Combining this with the third of the first-order conditions, we get

$$90 = 20X + 10(2X)$$

$$90 = 40X$$

$$X = 2.25$$

$$Y = 2(2.25) = 4.5$$

Carmen should buy 2.25 units of X and 4.5 units of Y at these prices. The total effect of the price change therefore is a decrease of 0.25 units of good X and no change in good Y.

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e. To decompose this total effect into income and substitution effects, we can solve the expenditure minimization problem $\min_{X,Y} 20X + 10Y$ s.t. $56.25 = 5XY$. The Lagrangian corresponding to this is

$$\mathcal{L}(X,Y,\lambda) = 20X + 10Y + \lambda(56.25 - 5XY)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = 20 - \lambda(5Y) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 10 - \lambda(5X) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 56.25 - 5XY = 0$$

This is now a system of three equations with three unknowns that can be solved for X, Y, and λ. X and Y give the allocation that will allow us to separate the substitution and income effects:

$$\lambda = \frac{10}{5X} = \frac{20}{5Y}$$

Therefore, $2X = Y$.

Combining this with the third of the first-order conditions, we get

$$56.25 = 5X(2X)$$

$$56.25 = 10X^2$$

$$X \approx 2.37$$

$$Y \approx 2(2.37) = 4.74$$

The substitution effect for good X therefore is the difference between 2.5 and 2.37 (representing a decrease of 0.13 units), and the income effect for good X is $2.25 - 2.37 = -0.12$ units. The total effect therefore is a decrease of 0.25 units, as in part (d). For good Y, we find that the substitution effect is $4.74 - 4.5 = 0.24$ units, and the income effect is $4.5 - 4.74 = -0.24$; hence, the total effect is zero.

f. Good X is a normal good for Carmen. As the price here increases (and therefore as Carmen experiences a drop in purchasing power), she decreases her consumption of good X (the income effect). Another way to answer is to note that the income and substitution effects for good X (the good experiencing a price change) are moving in the same direction.

g. Carmen's constrained optimization problem is $\max_{X,Y} 5XY$ s.t. $\overline{I} - p_X X + \overline{p_Y} Y$. The Lagrangian corresponding to this is

$$\max_{X,Y,\lambda} \mathcal{L}(X,Y,\lambda) = 5XY + \lambda(\overline{I} - p_X X - \overline{p_Y} Y)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = 5Y - \lambda(p_X) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = 5X - \lambda(\overline{p_Y}) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{I} - p_X X - \overline{p_Y} Y = 0$$

Combining the first two equations, we see that

$$\lambda = \frac{5Y}{p_X} = \frac{5X}{\overline{p_Y}}$$

Therefore, $\overline{p_Y} Y = p_X X$.

Combining this with the third of the first-order conditions, we get

$$\overline{I} = p_X X + \overline{p_Y} Y$$

$$\overline{I} = p_X X + p_X X$$

Carmen's Marshallian demand curve for X is $\frac{\overline{I}}{2p_X}$. Since $\frac{\partial X}{\partial p_X} = -\frac{\overline{I}}{2p_X^2} < 0$, the Law of Demand stating that demand is decreasing in price holds.

h. Carmen's constrained optimization problem is $\min_{X,Y} p_X X + \overline{p}_Y Y$ s.t. $\overline{U} = 5XY$. The Lagrangian corresponding to this is

$$\min_{X,Y,\lambda} \mathcal{L}(X,Y,\lambda) = p_X X + \overline{p_Y} Y + \lambda(\overline{U} - 5XY)$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial X} = p_X - \lambda(5Y) = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \overline{p_Y} - \lambda(5X) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{U} - 5X\text{Y} = 0$$

Combining the first two equations, we see that

$$\lambda = \frac{p_X}{5Y} = \frac{\overline{p_Y}}{5X}$$

Therefore, $\overline{p_Y}Y = p_X X$ or $Y = \frac{p_X X}{\overline{p_Y}}$.

Combining this with the third of the first-order conditions, we get

$$\overline{U} = 5XY$$

$$= 5X\left(\frac{p_X X}{\overline{p_Y}}\right)$$

Carmen's Hicksian demand curve X then is

$$X = \sqrt{\overline{U}\left(\frac{\overline{p_Y}}{5p_X}\right)}$$