Solution

6. a. Labor and capital display diminishing marginal products:

$$MP_K = 2\left(\frac{L}{K}\right)^{0.5}$$

As K increases, MP_K decreases. Similarly,

$$MP_L = 2\left(\frac{K}{L}\right)^{0.5}$$

MP_L is a decreasing function of L. The production function displays a diminishing marginal rate of technical substitution, since

$$MRTS = \frac{MP_L}{MP_K} = \frac{K}{L}$$

Consequently, as L increases, $MRTS$ decreases.

b. In this case, labor and capital do not display diminishing marginal returns. Since

$$MP_K = 4L$$

then as K increases, MP_K remains unchanged. Since

$$MP_L = 4K$$

then as L increases, MP_L remains unchanged.

The production function indeed displays a diminishing marginal rate of technical substitution. Because

$$MRTS = \frac{MP_L}{MP_K} = \frac{K}{L}$$

then as L increases, $MRTS$ decreases. Notice that the $MRTS$ for this production function is identical to the $MRTS$ of the prior function.

c. Labor and capital do not need to display diminishing marginal products in order for the $MRTS$ to diminish.

d. Marginal products can be calculated as partial derivatives of the production function. Since $Q = 4K^{0.5}L^{0.5}$, the marginal product of K is $MP_K = \frac{\partial Q}{\partial K} = 4(0.5)K^{0.5-1}L^{0.5} = 2K^{-0.5}L^{0.5}$, and the marginal product of L is $MP_L = \frac{\partial Q}{\partial L} = 4(0.5)K^{0.5}L^{0.5-1} = 2K^{0.5}L^{-0.5}$.

e. To see if labor and capital display diminishing marginal products, we can take the partial derivatives of the marginal products of labor and of capital with respect to their own input and check that they are negative. Specifically, $\frac{\partial MP_L}{\partial L} = 2(-0.5)K^{0.5}L^{-1.5} < 0$ and $\frac{\partial MP_K}{\partial K} = 2(-0.5)K^{-1.5}L^{0.5} < 0$. Yes, both display diminishing marginal products.

Since $MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{2K^{0.5}L^{-0.5}}{2K^{-0.5}L^{0.5}} = \frac{K}{L}$, we can take the partial derivative of this with respect to L (our x-axis input) and check that it is negative to see if the production function displays a diminishing marginal rate of technical substitution. Specifically, $\frac{\partial MRTS_{LK}}{\partial L} = -\frac{K}{L^2} < 0$. Yes, this production function has a diminishing marginal rate of substitution.

f. Marginal products can be calculated as partial derivatives of the production function. Since $Q = 4KL$, the marginal product of K is $MP_K = \frac{\partial Q}{\partial K} = 4L$, and the marginal product of L is $MP_L = \frac{\partial Q}{\partial L} = 4K$.

g. To see if labor and capital display diminishing marginal products, we can take the partial derivatives of the marginal products of labor and of capital with respect to their own input and check that they are negative. Specifically, $\frac{\partial MP_L}{\partial L} = 0$ and $\frac{\partial MP_K}{\partial K} = 0$. Here, therefore, we do not have diminishing marginal products. Instead, they are constant. Since $MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{4K}{4L} = \frac{K}{L}$ as in the other part of the problem, we know that this production function does have a diminishing marginal rate of technical substitution by our answer to part (e) above.