6.1 figure it out

The short-run production function for a firm that produces pizzas is $Q = f(\overline{K},L) = 15\overline{K}^{0.25}L^{0.75}$, where Q is the number of pizzas produced per hour, $\overline{K}$ is the number of ovens (which is fixed at 3 in the short run), and L is the number of workers employed.

a. Write an equation for the short-run production function for the firm showing output as a function of labor.

b. Calculate the total output produced per hour for $L = 0, 1, 2, 3, 4$, and 5.

c. Calculate the MP_L for $L = 1$ to $L = 5$. Is MP_L diminishing?

d. Calculate the AP_L for $L = 1$ to $L = 5$.

e. Use calculus to determine a function for the short-run marginal product of labor.

Solution:

a. To write the production function for the short run, we plug $\overline{K} = 3$ into the production function to create an equation that shows output as a function of labor:

$$Q = f(\overline{K},L) = 15\overline{K}^{0.25}L^{0.75}$$
$$= 15(3^{0.25})L^{0.75} = 15(1.316)L^{0.75} = 19.74L^{0.75}$$

b. To calculate total output, we plug in the different values of L and solve for Q:

$L = 0$	$Q = 19.74(0)^{0.75} = 19.74(0) = 0$
$L = 1$	$Q = 19.74(1)^{0.75} = 19.74(1) = 19.74$
$L = 2$	$Q = 19.74(2)^{0.75} = 19.74(1.682) = 33.20$
$L = 3$	$Q = 19.74(3)^{0.75} = 19.74(2.280) = 45.01$
$L = 4$	$Q = 19.74(4)^{0.75} = 19.74(2.828) = 55.82$
$L = 5$	$Q = 19.74(5)^{0.75} = 19.74(3.344) = 66.01$

c. The marginal product of labor is the additional output generated by an additional unit of labor, holding capital constant. We can use our answer from (b) to calculate the marginal product of labor for each worker:

$L = 1$	$MP_L = 19.74 - 0 = 19.74$
$L = 2$	$MP_L = 33.20 - 19.74 = 13.46$
$L = 3$	$MP_L = 45.01 - 33.20 = 11.81$
$L = 4$	$MP_L = 55.82 - 45.01 = 10.81$
$L = 5$	$MP_L = 66.01 - 55.82 = 10.19$

Note that, because MP_L falls as L rises, there is a diminishing marginal product of labor. This implies that output rises at a decreasing rate when labor is added to the fixed level of capital.

d. The average product of labor is calculated by dividing total output (Q) by the quantity of labor input (L):

$L = 1$	$AP_L = 19.74/1 = 19.74$
$L = 2$	$AP_L = 33.20/2 = 16.60$
$L = 3$	$AP_L = 45.01/3 = 15.00$
$L = 4$	$AP_L = 55.82/4 = 13.96$
$L = 5$	$AP_L = 66.01/5 = 13.20$

e. Marginal product of labor is the partial derivative of the production function with respect to labor. For the short run:

$$MP_L = \frac{\partial Q}{\partial L} = 15(0.75)\overline{K}^{0.25}L^{0.75-1} = 11.25\overline{K}^{0.25}L^{-0.25}.$$