Solution

7. a. Hack's fixed cost is $1,000.

b. Hack's short-run average variable cost of producing berries is

$$AVC = \frac{VC}{Q} = Q^2 - 12Q + 100$$

c. Hack will only operate in the short run when the market price is greater than or equal to its average variable cost curve (AVC) at the optimal level of output; the perfectly competitive short-run supply curve is the portion of the marginal cost curve MC above AVC. At prices below the AVC, the firm shuts down; that is, the quantity supplied is 0. To find the minimum price at which Hack is willing to produce, we equalize the marginal cost with the average variable cost so that

$$\begin{aligned} Q^2 - 12Q + 100 &= 3Q^2 - 24Q + 100 \\ 2Q(Q - 6) &= 0 \\ Q &= 6 \end{aligned}$$

The AVC at $Q = 6$ is

$$AVC = 6^2 - 12 \times 6 + 100 = \$64$$

Thus, if the price is $60, then Hack will not produce any berries, since that price lies below the minimum point of the AVC curve.

d. Yes. A price of $73 is above the $64 threshold, so Hack should produce a positive number of berries. However, that is only the case for the short run. Since Hack is earning a negative profit once fixed cost is taken into account, he should shut down in the long run.

e. Marginal cost is the derivative of the total cost function with respect to quantity:

$$\begin{aligned} MC &= \frac{dTC}{dQ} \\ &= 3Q^{3-1} - 12(2)Q^{2-1} + 100 + 0 \\ &= 3Q^2 - 24Q + 100 \end{aligned}$$

f. Average variable cost is shown in part (b): $AVC = Q^2 - 12Q + 100$. We can minimize this directly by calculating the first-order condition. Taking the derivative of AVC with respect to Q, we see that

$$\frac{dAVC}{dQ} = 2Q^{2-1} - 12 + 0 = 2Q - 12$$

Setting this equal to zero, $2Q - 12 = 0$ or $Q = 6$.

To confirm that this is a minimum (and not a maximum), we can check the second-order condition: $\frac{d^2AVC}{dQ^2} = 2 > 0$. Since this is positive, we know that we have a minimum. The firm should shut down if price is less than AVC at a quantity of 6: if $P < (6)^2 - 12(6) + 100 = \$64$.