∂ 8.2 figure it out

Cardboard boxes are produced in a perfectly competitive market. Each identical firm has a short-run total cost curve of $TC = 3Q^3 - 18Q^2 + 30Q + 50$, where Q is measured in thousands of boxes per week. The firm's associated marginal cost curve is $MC = 9Q^2 - 36Q + 30$.

a. Calculate the price below which a firm in the market will not produce any output in the short run (the shut-down price).

b. Show that marginal cost is as given using calculus.

c. Minimize average variable cost using calculus to derive the shut-down price and confirm that your answer is the same as in part (a).

Solution:

a. A firm will not produce any output in the short run at any price below its minimum AVC. How do we find the minimum AVC? We learned in Chapter 7 that AVC is minimized when $AVC = MC$. So, we need to start by figuring out the equation for the average variable cost curve and then solving it for the output that minimizes AVC.

AVC is equal to VC/Q. Remember that total cost is the sum of fixed cost and variable cost:

$$TC = FC + VC$$

Fixed cost is that part of total cost that does not vary with output (changes in Q have no effect on FC). Therefore, if $TC = 3Q^3 - 18Q^2 + 30Q + 50$, then FC must be 50. This means that $VC = 3Q^3 - 18Q^2 + 30Q$. Because $AVC = VC/Q$,

$$AVC = VC/Q = \frac{3Q^3 - 18Q^2 + 30Q}{Q} = 3Q^2 - 18Q + 30$$

Next, we find the output for which AVC is at its minimum by equating AVC and MC:

$$\begin{aligned} AVC &= MC \\ 3Q^2 - 18Q + 30 &= 9Q^2 - 36Q + 30 \\ 18Q &= 6Q^2 \\ 18 &= 6Q \\ Q &= 3 \end{aligned}$$

This means that AVC is at its minimum at an output of 3,000 cardboard boxes per week. To find the level of AVC at this output, we plug $Q = 3$ into the formula for AVC:

$$\begin{aligned} AVC &= 3Q^2 - 18Q + 30 \\ &= 3(3)^2 - 18(3) + 30 = 27 - 54 + 30 = \$3 \end{aligned}$$

Therefore, the minimum price at which the firm should operate is \$3. If the price falls below \$3, the firm should shut down in the short run and only pay its fixed cost.

b. Marginal cost is the derivative of the total cost function with respect to quantity:

$$\begin{aligned} MC &= \frac{dTC}{dQ} \\ &= 3(3)Q^{3-1} - 18(2)Q^{2-1} + 30 + 0 \\ &= 9Q^2 - 36Q + 30 \end{aligned}$$

This is the same equation for marginal cost as given in the problem.

c. Average variable cost is shown in part (a): $AVC = 3Q^2 - 18Q + 30$. We can minimize this directly by calculating the first-order condition.

Taking the derivative of AVC with respect to Q, we see that

$$\frac{dAVC}{dQ} = 3(2)Q^{2-1} - 18 + 0$$
$$= 6Q - 18$$

Setting this equal to zero, $6Q - 18 = 0$ or $Q = 3$.

To confirm that this is a minimum (and not a maximum), we can check the second-order condition: $\frac{d^2AVC}{dQ^2} = 6 > 0$. Since this is positive, we know that we have a minimum. This is the same quantity as found in part (a), and therefore the shut-down price is also the same. The firm should shut down if price is less than AVC at a quantity of 3: if $P < 3(3)^2 - 18(3) + 30 = \$3$.